CHAPTER 1

ENGINEERING MATHEMATICS

YEAR 2012 **ONE MARK**

MCQ 1.1 The area enclosed between the straight line $y = x$ and the parabola $y = x^2$ in the x-y plane is

(A) 1/6 (B) 1/4

(C) 1/3 (D) 1/2

MCQ 1.2 Consider the function $f(x) = |x|$ in the interval $-1 \le x \le 1$. At the point $x = 0$, $f(x)$ is

(A) continuous and differentiable

(B) non-continuous and differentiable

(C) continuous and non-differentiable

(D) neither continuous nor differentiable

MCQ 1.3 $\lim_{x \to 0}\left(\frac{1 - \cos x}{x^2}\right)$ is

(A) 1/4 (B) 1/2

(C) 1 (D) 2

MCQ 1.4 At $x = 0$, the function $f(x) = x^3 + 1$ has

(A) a maximum value (B) a minimum value

(C) a singularity (D) a point of inflection

MCQ 1.5 For the spherical surface $x^2 + y^2 + z^2 = 1$, the unit outward normal vector at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is given by

(A) $\frac{1}{\sqrt{2}}\boldsymbol{i} + \frac{1}{\sqrt{2}}\boldsymbol{j}$ (B) $\frac{1}{\sqrt{2}}\boldsymbol{i} - \frac{1}{\sqrt{2}}\boldsymbol{j}$

(C) $\boldsymbol{k}$ (D) $\frac{1}{\sqrt{3}}\boldsymbol{i} + \frac{1}{\sqrt{3}}\boldsymbol{j} + \frac{1}{\sqrt{3}}\boldsymbol{k}$

YEAR 2012 **TWO MARKS**

MCQ 1.6 The inverse Laplace transform of the function $F(s) = \frac{1}{s(s+1)}$ is given by

(A) $f(t) = \sin t$

(B) $f(t) = e^{-t}\sin t$

(C) $f(t) = e^{-t}$

(D) $f(t) = 1 - e^{-t}$

MCQ 1.7 For the matrix $\boldsymbol{A} = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, ONE of the normalized eigen vectors given as

(A) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$

(B) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(C) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$

(D) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

MCQ 1.8 A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black balls is

(A) 1/20

(B) 1/12

(C) 3/10

(D) 1/2

MCQ 1.9 Consider the differential equation $x^2(d^2y/dx^2) + x(dy/dx) - 4y = 0$ with the boundary conditions of $y(0) = 0$ and $y(1) = 1$. The complete solution of the differential equation is

(A) x^2

(B) $\sin\left(\frac{\pi x}{2}\right)$

(C) $e^x \sin\left(\frac{\pi x}{2}\right)$

(D) $e^{-x}\sin\left(\frac{\pi x}{2}\right)$

MCQ 1.10

$$\begin{aligned} x + 2y + z &= 4 \\ 2x + y + 2z &= 5 \\ x - y + z &= 1 \end{aligned}$$

The system of algebraic equations given above has

(A) a unique solution of $x = 1, y = 1$ and $z = 1$.

(B) only the two solutions of $(x = 1, y = 1, z = 1)$ and $(x = 2, y = 1, z = 0)$

(C) infinite number of solutions

(D) no feasible solution

YEAR 2011 **ONE MARK**

MCQ 1.11 A series expansion for the function $\sin\theta$ is

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(A) $1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-...$

(B) $\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-...$

(C) $1+\theta+\frac{\theta^2}{2!}+\frac{\theta^3}{3!}+...$

(D) $\theta+\frac{\theta^3}{3!}+\frac{\theta^5}{5!}+...$

MCQ 1.12 What is $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}$ equal to ?

(A) θ

(B) $\sin\theta$

(C) 0

(D) 1

MCQ 1.13 Eigen values of a real symmetric matrix are always

(A) positive

(B) negative

(C) real

(D) complex

MCQ 1.14 The product of two complex numbers $1+i$ and $2-5i$ is

(A) $7-3i$

(B) $3-4i$

(C) $-3-4i$

(D) $7+3i$

MCQ 1.15 If $f(x)$ is an even function and a is a positive real number, then $\int_{-a}^{a} f(x)\,dx$ equals

(A) 0

(B) a

(C) $2a$

(D) $2\int_0^a f(x)\,dx$

YEAR 2011 TWO MARKS

MCQ 1.16 The integral $\int_1^3 \frac{1}{x}dx$, when evaluated by using Simpson's 1/3 rule on two equal sub-intervals each of length 1, equals

(A) 1.000

(B) 1.098

(C) 1.111

(D) 1.120

MCQ 1.17 Consider the differential equation $\frac{dy}{dx}=(1+y^2)x$. The general solution with constant c is

(A) $y=\tan\frac{x^2}{2}+\tan c$

(B) $y=\tan^2\left(\frac{x}{2}+c\right)$

(C) $y=\tan^2\left(\frac{x}{2}\right)+c$

(D) $y=\tan\left(\frac{x^2}{2}+c\right)$

MCQ 1.18 An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail. The probability of getting at least one head is

(A) $\frac{1}{32}$

(B) $\frac{13}{32}$

(C) $\frac{16}{32}$

(D) $\frac{31}{32}$

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MCQ 1.19 Consider the following system of equations

$$2x_1 + x_2 + x_3 = 0$$
$$x_2 - x_3 = 0$$
$$x_1 + x_2 = 0$$

This system has

(A) a unique solution

(B) no solution

(C) infinite number of solutions

(D) five solutions

YEAR 2010 ONE MARK

MCQ 1.20 The parabolic arc $y = \sqrt{x}$, $1 \le x \le 2$ is revolved around the x-axis. The volume of the solid of revolution is

(A) $\pi/4$

(B) $\pi/2$

(C) $3\pi/4$

(D) $3\pi/2$

MCQ 1.21 The Blasius equation, $\frac{d^3 f}{d\eta^3} + \frac{f}{2}\frac{d^2 f}{d\eta^2} = 0$, is a

(A) second order nonlinear ordinary differential equation

(B) third order nonlinear ordinary differential equation

(C) third order linear ordinary differential equation

(D) mixed order nonlinear ordinary differential equation

MCQ 1.22 The value of the integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ is

(A) $-\pi$ (B) $-\pi/2$

(C) $\pi/2$ (D) π

MCQ 1.23 The modulus of the complex number $\left(\frac{3+4i}{1-2i}\right)$ is

(A) 5 (B) $\sqrt{5}$

(C) $1/\sqrt{5}$ (D) $1/5$

MCQ 1.24 The function $y = |2 - 3x|$

(A) is continuous $\forall x \in R$ and differentiable $\forall x \in R$

(B) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = 3/2$

(C) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = 2/3$

(D) is continuous $\forall x \in R$ except $x = 3$ and differentiable $\forall x \in R$

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YEAR 2010 TWO MARKS

MCQ 1.25 One of the eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is

(A) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$
(B) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
(C) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$
(D) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

MCQ 1.26 The Laplace transform of a function $f(t)$ is $\frac{1}{s^2(s+1)}$. The function $f(t)$ is

(A) $t - 1 + e^{-t}$
(B) $t + 1 + e^{-t}$
(C) $-1 + e^{-t}$
(D) $2t + e^{t}$

MCQ 1.27 A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is

(A) 2/315
(B) 1/630
(C) 1/1260
(D) 1/2520

MCQ 1.28 Torque exerted on a flywheel over a cycle is listed in the table. Flywheel energy (in J per unit cycle) using Simpson's rule is

Angle (Degree)	0	60°	120°	180°	240°	300°	360°
Torque (N-m)	0	1066	−323	0	323	−355	0

(A) 542
(B) 993
(C) 1444
(D) 1986

YEAR 2009 ONE MARK

MCQ 1.29 For a matrix $[M] = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by

(A) $-\frac{4}{5}$
(B) $-\frac{3}{5}$
(C) $\frac{3}{5}$
(D) $\frac{4}{5}$

MCQ 1.30 The divergence of the vector field $3xz\boldsymbol{i} + 2xy\boldsymbol{j} - yz^2\boldsymbol{k}$ at a point $(1,1,1)$ is equal to

(A) 7
(B) 4
(C) 3
(D) 0

MCQ 1.31 The inverse Laplace transform of $1/(s^2 + s)$ is

(A) $1 + e^t$ (B) $1 - e^t$

(C) $1 - e^{-t}$ (D) $1 + e^{-t}$

MCQ 1.32 If three coins are tossed simultaneously, the probability of getting at least one head is

(A) 1/8 (B) 3/8

(C) 1/2 (D) 7/8

YEAR 2009 **TWO MARKS**

MCQ 1.33 An analytic function of a complex variable $z = x + iy$ is expressed as $f(z) = u(x,y) + iv(x,y)$ where $i = \sqrt{-1}$. If $u = xy$, the expression for v should be

(A) $\frac{(x+y)^2}{2} + k$ (B) $\frac{x^2 - y^2}{2} + k$

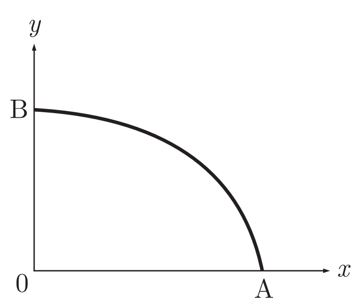
(C) $\frac{y^2 - x^2}{2} + k$ (D) $\frac{(x-y)^2}{2} + k$

MCQ 1.34 The solution of $x\frac{dy}{dx} + y = x^4$ with the condition $y(1) = \frac{6}{5}$ is

(A) $y = \frac{x^4}{5} + \frac{1}{x}$ (B) $y = \frac{4x^4}{5} + \frac{4}{5x}$

(C) $y = \frac{x^4}{5} + 1$ (D) $y = \frac{x^5}{5} + 1$

MCQ 1.35 A path AB in the form of one quarter of a circle of unit radius is shown in the figure. Integration of $(x+y)^2$ on path AB traversed in a counter-clockwise sense is



(A) $\frac{\pi}{2} - 1$ (B) $\frac{\pi}{2} + 1$

(C) $\frac{\pi}{2}$ (D) 1

MCQ 1.36 The distance between the origin and the point nearest to it on the surface $z^2 = 1 + xy$ is

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(A) 1 (B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$ (D) 2

MCQ 1.37 The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is

(A) $\frac{16}{3}$ (B) 8

(C) $\frac{32}{3}$ (D) 16

MCQ 1.38 The standard deviation of a uniformly distributed random variable between 0 and 1 is

(A) $\frac{1}{\sqrt{12}}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{5}{\sqrt{12}}$ (D) $\frac{7}{\sqrt{12}}$

YEAR 2008 **ONE MARK**

MCQ 1.39 In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x - 2)^4$ is

(A) 1/4 ! (B) $2^4/4!$

(C) $e^2/4!$ (D) $e^4/4!$

MCQ 1.40 Given that $\ddot{x} + 3x = 0$, and $x(0) = 1$, $\dot{x}(0) = 0$, what is $x(1)$?

(A) -0.99 (B) -0.16

(C) 0.16 (D) 0.99

MCQ 1.41 The value of $\lim_{x \to 8} \frac{x^{1/3} - 2}{(x - 8)}$

(A) $\frac{1}{16}$ (B) $\frac{1}{12}$

(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

MCQ 1.42 A coin is tossed 4 times. What is the probability of getting heads exactly 3 times ?

(A) $\frac{1}{4}$ (B) $\frac{3}{8}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

MCQ 1.43 The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigen value equal to 3. The sum of the other two eigen value is

(A) p (B) $p - 1$

(C) $p - 2$ (D) $p - 3$

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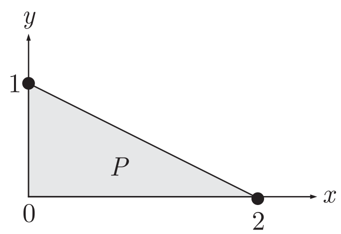
MCQ 1.44 The divergence of the vector field $(x-y)\boldsymbol{i}+(y-x)\boldsymbol{j}+(x+y+z)\boldsymbol{k}$ is

(A) 0 (B) 1

(C) 2 (D) 3

YEAR 2008 TWO MARKS

MCQ 1.45 Consider the shaded triangular region P shown in the figure. What is $\iint_P xy\,dx\,dy$?



(A) $\frac{1}{6}$ (B) $\frac{2}{9}$

(C) $\frac{7}{16}$ (D) 1

MCQ 1.46 The directional derivative of the scalar function $f(x,y,z)=x^2+2y^2+z$ at the point

$P=(1,1,2)$ in the direction of the vector $\boldsymbol{a}=3\boldsymbol{i}-4\boldsymbol{j}$ is

(A) -4 (B) -2

(C) -1 (D) 1

MCQ 1.47 For what value of a, if any will the following system of equation in x, y and z have a solution ?

$$2x+3y=4$$
$$x+y+z=4$$
$$3x+2y-z=a$$

(A) Any real number (B) 0

(C) 1 (D) There is no such value

MCQ 1.48 Which of the following integrals is unbounded ?

(A) $\int_0^{\pi/4}\tan x\,dx$ (B) $\int_0^{\infty}\frac{1}{x^2+1}dx$

(C) $\int_0^{\infty}xe^{-x}dx$ (D) $\int_0^{1}\frac{1}{1-x}dx$

MCQ 1.49 The integral $\oint f(z)\,dz$ evaluated around the unit circle on the complex plane

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for $f(z) = \frac{\cos z}{z}$ is

(A) $2\pi i$ (B) $4\pi i$

(C) $-2\pi i$ (D) 0

MCQ 1.50 The length of the curve $y = \frac{2}{3}x^{3/2}$ between $x = 0$ and $x = 1$ is

(A) 0.27 (B) 0.67

(C) 1 (D) 1.22

MCQ 1.51 The eigen vector of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is $a + b$?

(A) 0 (B) $\frac{1}{2}$

(C) 1 (D) 2

MCQ 1.52 Let $f = y^x$. What is $\frac{\partial^2 f}{\partial x \partial y}$ at $x = 2, y = 1$?

(A) 0 (B) $\ln 2$

(C) 1 (D) $\frac{1}{\ln 2}$

MCQ 1.53 It is given that $y'' + 2y' + y = 0$, $y(0) = 0, y(1) = 0$. What is $y(0.5)$?

(A) 0 (B) 0.37

(C) 0.62 (D) 1.13

YEAR 2007 ONE MARK

MCQ 1.54 The minimum value of function $y = x^2$ in the interval [1, 5] is

(A) 0 (B) 1

(C) 25 (D) undefined

MCQ 1.55 If a square matrix A is real and symmetric, then the eigen values

(A) are always real (B) are always real and positive

(C) are always real and non-negative pairs (D) occur in complex conjugate

MCQ 1.56 If $\varphi(x, y)$ and $\psi(x, y)$ are functions with continuous second derivatives, then $\varphi(x, y) + i\psi(x, y)$ can be expressed as an analytic function of $x + i\psi (i = \sqrt{-1})$, when

(A) $\frac{\partial \varphi}{\partial x} = -\frac{\partial \psi}{\partial x}, \frac{\partial \varphi}{\partial y} = \frac{\partial \psi}{\partial y}$ (B) $\frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x}, \frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y}$

(C) $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 1$ (D) $\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} = \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = 0$

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MCQ 1.57 The partial differential equation $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} = 0$ has

(A) degree 1 order 2
(B) degree 1 order 1
(C) degree 2 order 1
(D) degree 2 order 2

YEAR 2007 **TWO MARKS**

MCQ 1.58 If $y = x + \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$, then $y(2) =$

(A) 4 or 1
(B) 4 only
(C) 1 only
(D) undefined

MCQ 1.59 The area of a triangle formed by the tips of vectors $\bar{a}, \bar{b}$ and $\bar{c}$ is

(A) $\frac{1}{2}(\boldsymbol{a} - \boldsymbol{b}) \cdot (\boldsymbol{a} - \boldsymbol{c})$
(B) $\frac{1}{2}|(\boldsymbol{a} - \boldsymbol{b}) \times (\boldsymbol{a} - \boldsymbol{c})|$
(C) $\frac{1}{2}|\boldsymbol{a} \times \boldsymbol{b} \times \boldsymbol{c}|$
(D) $\frac{1}{2}(\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c}$

MCQ 1.60 The solution of $\frac{dy}{dx} = y^2$ with initial value $y(0) = 1$ bounded in the interval

(A) $-\infty \leq x \leq \infty$
(B) $-\infty \leq x \leq 1$
(C) $x < 1, x > 1$
(D) $-2 \leq x \leq 2$

MCQ 1.61 If $F(s)$ is the Laplace transform of function $f(t)$, then Laplace transform of $\int_0^t f(\tau)\, d\tau$ is

(A) $\frac{1}{s}F(s)$
(B) $\frac{1}{s}F(s) - f(0)$
(C) $sF(s) - f(0)$
(D) $\int F(s)\, ds$

MCQ 1.62 A calculator has accuracy up to 8 digits after decimal place. The value of

$$\int_0^{2\pi} \sin x dx$$

when evaluated using the calculator by trapezoidal method with 8 equal intervals, to 5 significant digits is

(A) 0.00000
(B) 1.0000
(C) 0.00500
(D) 0.00025

MCQ 1.63 Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE ?

(A) $E(XY) = E(X)E(Y)$
(B) $\text{Cov}(X, Y) = 0$
(C) $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$
(D) $E(X^2 Y^2) = (E(X))^2 (E(Y))^2$

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MCQ 1.64 $\lim_{x \to 0} \frac{e^x - \left(1 + x + \frac{x^2}{2}\right)}{x^3} =$

(A) 0
(B) 1/6
(C) 1/3
(D) 1

MCQ 1.65 The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is

(A) 0
(B) 1
(C) 2
(D) infinite

YEAR 2006 ONE MARK

MCQ 1.66 Match the items in column I and II.

	Column I		**Column II**
P.	Gauss-Seidel method	**1.**	Interpolation
Q.	Forward Newton-Gauss method	**2.**	Non-linear differential equations
R.	Runge-Kutta method	**3.**	Numerical integration
S.	Trapezoidal Rule	**4.**	Linear algebraic equations

(A) P-1, Q-4, R-3, S-2
(B) P-1, Q-4, R-2, S-3
(C) P-1. Q-3, R-2, S-4
(D) P-4, Q-1, R-2, S-3

MCQ 1.67 The solution of the differential equation $\frac{dy}{dx} + 2xy = e^{-x^2}$ with $y(0) = 1$ is

(A) $(1 + x)\, e^{+x^2}$
(B) $(1 + x)\, e^{-x^2}$
(C) $(1 - x)\, e^{+x^2}$
(D) $(1 - x)\, e^{-x^2}$

MCQ 1.68 Let x denote a real number. Find out the INCORRECT statement.

(A) $S = \{x : x > 3\}$ represents the set of all real numbers greater than 3
(B) $S = \{x : x^2 < 0\}$ represents the empty set.
(C) $S = \{x : x \in A \text{ and } x \in B\}$ represents the union of set A and set B.
(D) $S = \{x : a < x < b\}$ represents the set of all real numbers between a and b, where a and b are real numbers.

MCQ 1.69 A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective ?

(A) $\frac{1}{5}$
(B) $\frac{1}{25}$
(C) $\frac{20}{99}$
(D) $\frac{19}{495}$

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YEAR 2006 **TWO MARKS**

MCQ 1.70 Eigen values of a matrix $S=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$ are 5 and 1. What are the eigen values of the matrix $S^2 = SS$?

(A) 1 and 25
(B) 6 and 4
(C) 5 and 1
(D) 2 and 10

MCQ 1.71 Equation of the line normal to function $f(x) = (x-8)^{2/3}+1$ at $P(0,5)$ is

(A) $y = 3x-5$
(B) $y = 3x+5$
(C) $3y = x+15$
(D) $3y = x-15$

MCQ 1.72 Assuming $i=\sqrt{-1}$ and t is a real number, $\int_0^{\pi/3} e^{it}\,dt$ is

(A) $\frac{\sqrt{3}}{2}+i\frac{1}{2}$
(B) $\frac{\sqrt{3}}{2}-i\frac{1}{2}$
(C) $\frac{1}{2}+i\frac{\sqrt{3}}{2}$
(D) $\frac{1}{2}+i\left(1-\frac{\sqrt{3}}{2}\right)$

MCQ 1.73 If $f(x)=\frac{2x^2-7x+3}{5x^2-12x-9}$, then $\lim_{x\to 3} f(x)$ will be

(A) $-1/3$
(B) $5/18$
(C) 0
(D) $2/5$

MCQ 1.74 Match the items in column I and II.

	Column I		Column II
P.	Singular matrix	**1.**	Determinant is not defined
Q.	Non-square matrix	**2.**	Determinant is always one
R.	Real symmetric	**3.**	Determinant is zero
S.	Orthogonal matrix	**4.**	Eigenvalues are always real
		5.	Eigenvalues are not defined

(A) P-3, Q-1, R-4, S-2
(B) P-2, Q-3, R-4, S-1
(C) P-3, Q-2, R-5, S-4
(D) P-3, Q-4, R-2, S-1

MCQ 1.75 For $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=3e^{2x}$, the particular integral is

(A) $\frac{1}{15}e^{2x}$
(B) $\frac{1}{5}e^{2x}$
(C) $3e^{2x}$
(D) $C_1e^{-x}+C_2e^{-3x}$

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MCQ 1.76 Multiplication of matrices E and F is G. matrices E and G are

$$E=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \text{ and } G=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

What is the matrix F ?

(A) $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

(B) $\begin{bmatrix}\cos\theta & \cos\theta & 0\\ -\cos\theta & \sin\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

(C) $\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

(D) $\begin{bmatrix}\sin\theta & -\cos\theta & 0\\ \cos\theta & \sin\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

MCQ 1.77 Consider the continuous random variable with probability density function

$$f(t) = 1+t \text{ for } -1 \le t \le 0$$
$$= 1-t \text{ for } 0 \le t \le 1$$

The standard deviation of the random variable is

(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{1}{\sqrt{6}}$

(C) $\frac{1}{3}$

(D) $\frac{1}{6}$

YEAR 2005 ONE MARK

MCQ 1.78 Stokes theorem connects

(A) a line integral and a surface integral

(B) a surface integral and a volume integral

(C) a line integral and a volume integral

(D) gradient of a function and its surface integral

MCQ 1.79 A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is

(A) 0.0036

(B) 0.1937

(C) 0.2234

(D) 0.3874

MCQ 1.80 $\int_{-a}^{a}(\sin^6 x+\sin^7 x)\,dx$ is equal to

(A) $2\int_0^a \sin^6 x\,dx$

(B) $2\int_0^a \sin^7 x\,dx$

(C) $2\int_0^a (\sin^6 x+\sin^7 x)\,dx$

(D) zero

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MCQ 1.81 A is a 3×4 real matrix and $Ax = b$ is an inconsistent system of equations. The highest possible rank of A is

(A) 1 (B) 2

(C) 3 (D) 4

MCQ 1.82 Changing the order of the integration in the double integral $I = \int_0^8 \int_{\frac{x}{4}}^2 f(x,y)\,dy dx$ leads to $I = \int_r^s \int_p^q f(x,y)\,dx dy$ What is q ?

(A) $4y$ (B) $16\ y^2$

(C) x (D) 8

YEAR 2005 **TWO MARKS**

MCQ 1.83 Which one of the following is an eigen vector of the matrix

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

(A) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$

MCQ 1.84 With a 1 unit change in b, what is the change in x in the solution of the system of equations $x + y = 2, 1.01x + 0.99y = b$?

(A) zero (B) 2 units

(C) 50 units (D) 100 units

MCQ 1.85 By a change of variable $x(u,v) = uv, y(u,v) = v/u$ is double integral, the integrand $f(x,y)$ changes to $f(uv, v/u)\phi(u,v)$. Then, $\phi(u,v)$ is

(A) $2v/u$ (B) $2uv$

(C) v^2 (D) 1

MCQ 1.86 The right circular cone of largest volume that can be enclosed by a sphere of 1 m radius has a height of

(A) 1/3 m (B) 2/3 m

(C) $\frac{2\sqrt{2}}{3}$m (D) 4/3 m

MCQ 1.87 If $x^2\frac{dy}{dx}+2xy=\frac{2\ln(x)}{x}$ and $y(1)=0$, then what is $y(e)$?

(A) e
(B) 1
(C) $1/e$
(D) $1/e^2$

MCQ 1.88 The line integral $\int \boldsymbol{V}\cdot d\boldsymbol{r}$ of the vector $\boldsymbol{V}\cdot(\boldsymbol{r})=2xyz\boldsymbol{i}+x^2z\boldsymbol{j}+x^2y\boldsymbol{k}$ from the origin to the point P (1, 1, 1)

(A) is 1
(B) is zero
(C) is – 1
(D) cannot be determined without specifying the path

MCQ 1.89 Starting from $x_0=1$, one step of Newton-Raphson method in solving the equation $x^3+3x-7=0$ gives the next value (x_1) as

(A) $x_1=0.5$
(B) $x_1=1.406$
(C) $x_1=1.5$
(D) $x_1=2$

MCQ 1.90 A single die is thrown twice. What is the probability that the sum is neither 8 nor 9 ?

(A) 1/9
(B) 5/36
(C) 1/4
(D) 3/4

- **Common Data For Q. 91 and 92**

The complete solution of the ordinary differential equation

$$\frac{d^2y}{dx^2}+p\frac{dy}{dx}+qy=0 \text{ is } y=c_1e^{-x}+c_2e^{-3x}$$

MCQ 1.91 Then p and q are

(A) $p=3, q=3$
(B) $p=3, q=4$
(C) $p=4, q=3$
(D) $p=4, q=4$

MCQ 1.92 Which of the following is a solution of the differential equation

$$\frac{d^2y}{dx^2}+p\frac{dy}{dx}+(q+1)y=0$$

(A) e^{-3x}
(B) xe^{-x}
(C) xe^{-2x}
(D) x^2e^{-2x}

YEAR 2004 **ONE MARK**

MCQ 1.93 If $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$, then $\frac{dy}{dx}$ will be equal to

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(A) $\sin\left(\frac{\theta}{2}\right)$ (B) $\cos\left(\frac{\theta}{2}\right)$

(C) $\tan\left(\frac{\theta}{2}\right)$ (D) $\cot\left(\frac{\theta}{2}\right)$

MCQ 1.94 The angle between two unit-magnitude coplanar vectors $P(0.866, 0.500, 0)$ and $Q(0.259, 0.966, 0)$ will be

(A) $0°$ (B) $30°$

(C) $45°$ (D) $60°$

MCQ 1.95 The sum of the eigen values of the matrix given below is $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(A) 5 (B) 7

(C) 9 (D) 18

YEAR 2004 TWO MARKS

MCQ 1.96 From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be Kings, if first card in NOT replaced ?

(A) $\frac{1}{26}$ (B) $\frac{1}{52}$

(C) $\frac{1}{169}$ (D) $\frac{1}{221}$

MCQ 1.97 A delayed unit step function is defined as $U(t-a) = \begin{cases} 0, \text{for } t < a \\ 1, \text{for } t \geq a \end{cases}$ Its Laplace transform is

(A) ae^{-as} (B) $\frac{e^{-as}}{s}$

(C) $\frac{e^{as}}{s}$ (D) $\frac{e^{as}}{a}$

MCQ 1.98 The values of a function $f(x)$ are tabulated below

x	$f(x)$
0	1
1	2
2	1
3	10

Using Newton's forward difference formula, the cubic polynomial that can be fitted to the above data, is

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(A) $2x^3+7x^2-6x+2$
(B) $2x^3-7x^2+6x-2$
(C) $x^3-7x^2-6x^2+1$
(D) $2x^3-7x^2+6x+1$

MCQ 1.99 The volume of an object expressed in spherical co-ordinates is given by

$$V=\int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2\sin\phi\,dr\,d\phi\,d\theta$$

The value of the integral is

(A) $\frac{\pi}{3}$
(B) $\frac{\pi}{6}$
(C) $\frac{2\pi}{3}$
(D) $\frac{\pi}{4}$

MCQ 1.100 For which value of x will the matrix given below become singular ?

$$=\begin{bmatrix}8 & x & 0\\ 4 & 0 & 2\\ 12 & 6 & 0\end{bmatrix}$$

(A) 4
(B) 6
(C) 8
(D) 12

YEAR 2003 ONE MARK

MCQ 1.101 $\lim_{x\to 0}\frac{\sin^2 x}{x}$ is equal to

(A) 0
(B) ∞
(C) 1
(D) -1

MCQ 1.102 The accuracy of Simpson's rule quadrature for a step size h is

(A) $O(h^2)$
(B) $O(h^3)$
(C) $O(h^4)$
(D) $O(h^5)$

MCQ 1.103 For the matrix $\begin{bmatrix}4 & 1\\ 1 & 4\end{bmatrix}$ the eigen values are

(A) 3 and -3
(B) -3 and -5
(C) 3 and 5
(D) 5 and 0

YEAR 2003 TWO MARKS

MCQ 1.104 Consider the system of simultaneous equations

$$x+2y+z=6$$
$$2x+y+2z=6$$
$$x+y+z=5$$

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This system has

(A) unique solution

(B) infinite number of solutions

(C) no solution

(D) exactly two solutions

MCQ 1.105 The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ is

(A) 1/8 (B) 1/6

(C) 1/3 (D) 1/2

MCQ 1.106 The solution of the differential equation $\frac{dy}{dx} + y^2 = 0$ is

(A) $y = \frac{1}{x + c}$ (B) $y = \frac{-x^3}{3} + c$

(C) ce^x (D) unsolvable as equation is non-linear

MCQ 1.107 The vector field is $\boldsymbol{F} = x\boldsymbol{i} - y\boldsymbol{j}$ (where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vector) is

(A) divergence free, but not irrotational

(B) irrotational, but not divergence free

(C) divergence free and irrotational

(D) neither divergence free nor irrational

MCQ 1.108 Laplace transform of the function $\sin \omega t$ is

(A) $\frac{s}{s^2 + \omega^2}$ (B) $\frac{\omega}{s^2 + \omega^2}$

(C) $\frac{s}{s^2 - \omega^2}$ (D) $\frac{\omega}{s^2 - \omega^2}$

MCQ 1.109 A box contains 5 black and 5 red balls. Two balls are randomly picked one after another form the box, without replacement. The probability for balls being red is

(A) 1/90 (B) 1/2

(C) 19/90 (D) 2/9

YEAR 2002 ONE MARK

MCQ 1.110 Two dice are thrown. What is the probability that the sum of the numbers on the two dice is eight?

(A) $\frac{5}{36}$ (B) $\frac{5}{18}$

(C) $\frac{1}{4}$ (D) $\frac{1}{3}$

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MCQ 1.111 Which of the following functions is not differentiable in the domain $[-1,1]$?

(A) $f(x) = x^2$ (B) $f(x) = x - 1$

(C) $f(x) = 2$ (D) $f(x) =$ maximum $(x, -x)$

MCQ 1.112 A regression model is used to express a variable Y as a function of another variable X. This implies that

(A) there is a causal relationship between Y and X

(B) a value of X may be used to estimate a value of Y

(C) values of X exactly determine values of Y

(D) there is no causal relationship between Y and X

YEAR 2002 TWO MARKS

MCQ 1.113 The following set of equations has

$$3x + 2y + z = 4$$
$$x - y + z = 2$$
$$-2x + 2z = 5$$

(A) no solution (B) a unique solution

(C) multiple solutions (D) an inconsistency

MCQ 1.114 The function $f(x, y) = 2x^2 + 2xy - y^3$ has

(A) only one stationary point at $(0, 0)$

(B) two stationary points at $(0, 0)$ and $\left(\frac{1}{6}, \frac{-1}{3}\right)$

(C) two stationary points at $(0, 0)$ and $(1, -1)$

(D) no stationary point

MCQ 1.115 Manish has to travel from A to D changing buses at stops B and C enroute. The maximum waiting time at either stop can be 8 min each but any time of waiting up to 8 min is equally, likely at both places. He can afford up to 13 min of total waiting time if he is to arrive at D on time. What is the probability that Manish will arrive late at D ?

(A) $\frac{8}{13}$ (B) $\frac{13}{64}$

(C) $\frac{119}{128}$ (D) $\frac{9}{128}$

YEAR 2001 ONE MARK

MCQ 1.116 The divergence of vector $\boldsymbol{i} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ is

(A) $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ (B) 3

(C) 0 (D) 1

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MCQ 1.117 Consider the system of equations given below

$$x + y = 2$$
$$2x + 2y = 5$$

This system has

(A) one solution
(B) no solution
(C) infinite solutions
(D) four solutions

MCQ 1.118 What is the derivative of $f(x) = |x|$ at $x = 0$?

(A) 1
(B) -1
(C) 0
(D) Does not exist

MCQ 1.119 The Gauss divergence theorem relates certain

(A) surface integrals to volume integrals
(B) surface integrals to line integrals
(C) vector quantities to other vector quantities
(D) line integrals to volume integrals

YEAR 2001 TWO MARKS

MCQ 1.120 The minimum point of the function $f(x) = \left(\frac{x^3}{3}\right) - x$ is at

(A) $x = 1$
(B) $x = -1$
(C) $x = 0$
(D) $x = \frac{1}{\sqrt{3}}$

MCQ 1.121 The rank of a 3×3 matrix $C(= AB)$, found by multiplying a non-zero column matrix A of size 3×1 and a non-zero row matrix B of size 1×3, is

(A) 0
(B) 1
(C) 2
(D) 3

MCQ 1.122 An unbiased coin is tossed three times. The probability that the head turns up in exactly two cases is

(A) $\frac{1}{9}$
(B) $\frac{1}{8}$
(C) $\frac{2}{3}$
(D) $\frac{3}{8}$

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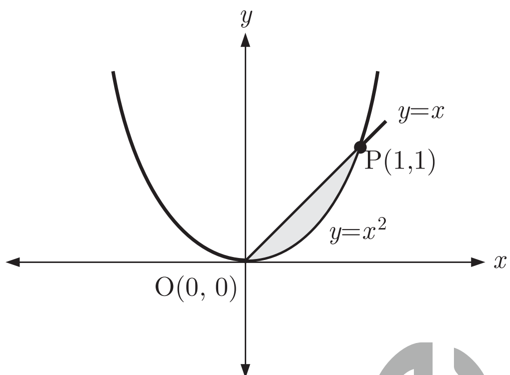
SOLUTION

SOL 1.1 Option (A) is correct.

For $y = x$ straight line and

$y = x^2$ parabola, curve is as given. The shaded region is the area, which is bounded by the both curves (common area).



We solve given equation as follows to gett the intersection points :

In $y = x^2$ putting $y = x$ we have $x = x^2$ or

$$x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0,1$$

Then from $y = x$, for $x = 0 \Rightarrow y = 0$ and $x = 1 \Rightarrow y = 1$

Curve $y = x^2$ and $y = x$ intersects at point (0,0) and (1,1)

So, the area bounded by both the curves is

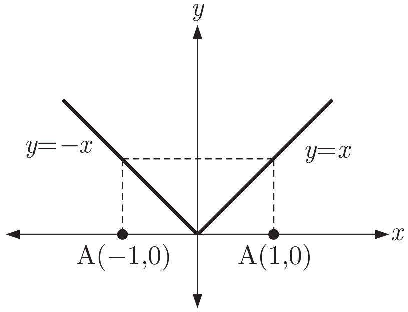
$$A = \int_{x=0}^{x=1}\int_{y=x}^{y=x^2} dydx = \int_{x=0}^{x=1} dx\int_{y=x}^{y=x^2} dy = \int_{x=0}^{x=1} dx[y]_x^{x^2} = \int_{x=0}^{x=1}(x^2 - x)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6} = \frac{1}{6}\text{unit}^2 \quad \text{Area is never negative}$$

SOL 1.2 Option (C) is correct.

Given $f(x) = |x|$ (in $-1 \le x \le 1$)

For this function the plot is as given below.



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At $x = 0$, function is continuous but not differentiable because.

For $x > 0$ and $x < 0$

$$f'(x) = 1 \text{ and } f'(x) = -1$$

$$\lim_{x \to 0^+} f'(x) = 1 \text{ and } \lim_{x \to 0^-} f'(x) = -1$$

R.H.S lim = 1 and L.H.S lim = − 1

Therefore it is not differentiable.

SOL 1.3 Option (B) is correct.

Let $y = \lim_{x \to 0} \frac{(1 - \cos x)}{x^2}$

It forms $\left[\frac{0}{0}\right]$ condition. Hence by L-Hospital rule

$$y = \lim_{x \to 0} \frac{\frac{d}{dx}(1 - \cos x)}{\frac{d}{dx}(x^2)} = \lim_{x \to 0} \frac{\sin x}{2x}$$

Still these gives $\left[\frac{0}{0}\right]$ condition, so again applying L-Hospital rule

$$y = \lim_{x \to 0} \frac{\frac{d}{dx}(\sin x)}{2 \times \frac{d}{dx}(x)} = \lim_{x \to 0} \frac{\cos x}{2} = \frac{\cos 0}{2} = \frac{1}{2}$$

SOL 1.4 Option (D) is correct.

We have $f(x) = x^3 + 1$

$$f'(x) = 3x^2 + 0$$

Putting $f'(x)$ equal to zero

$$f'(x) = 0$$

$$3x^2 + 0 = 0 \Rightarrow x = 0$$

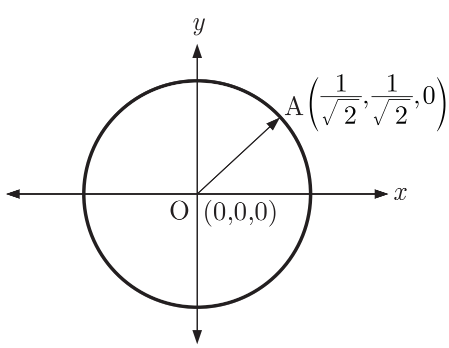
Now $f''(x) = 6x$

At $x = 0$, $f''(0) = 6 \times 0 = 0$ Hence $x = 0$ is the point of inflection.

SOL 1.5 Option (A) is correct.

Given : $x^2 + y^2 + z^2 = 1$

This is a equation of sphere with radius $r = 1$



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The unit normal vector at point $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)$ is $\boldsymbol{OA}$

Hence $\boldsymbol{OA} = \left(\frac{1}{\sqrt{2}}-0\right)\boldsymbol{i}+\left(\frac{1}{\sqrt{2}}-0\right)\boldsymbol{j}+(0-0)\,\boldsymbol{k} = \frac{1}{\sqrt{2}}\boldsymbol{i}+\frac{1}{\sqrt{2}}\boldsymbol{j}$

SOL 1.6 Option (D) is correct.

First using the partial fraction :

$$F(s) = \frac{1}{s(s+1)} = \frac{A}{s}+\frac{B}{s+1} = \frac{A(s+1)+Bs}{s(s+1)}$$

$$\frac{1}{s(s+1)} = \frac{(A+B)\,s}{s(s+1)}+\frac{A}{s(s+1)}$$

Comparing the coefficients both the sides,

$$(A+B) = 0 \text{ and } A = 1,\ B = -1$$

So $$\frac{1}{s(s+1)} = \frac{1}{s}-\frac{1}{s+1}$$

$$F(t) = L^{-1}[F(s)]$$

$$= L^{-1}\left[\frac{1}{s(s+1)}\right] = L^{-1}\left[\frac{1}{s}-\frac{1}{s+1}\right] = L^{-1}\left[\frac{1}{s}\right]-L^{-1}\left[\frac{1}{s+1}\right]$$

$$= 1-e^{-t}$$

SOL 1.7 Option (B) is correct.

Given $$\boldsymbol{A} = \begin{bmatrix}5 & 3\\1 & 3\end{bmatrix}$$

For finding eigen values, we write the characteristic equation as

$$|\boldsymbol{A}-\lambda\boldsymbol{I}| = 0$$

$$\begin{vmatrix}5-\lambda & 3\\1 & 3-\lambda\end{vmatrix} = 0$$

$\Rightarrow$ $$(5-\lambda)(3-\lambda)-3 = 0$$

$$\lambda^2-8\lambda+12 = 0 \Rightarrow \lambda = 2,6$$

Now from characteristic equation for eigen vector.

$$[\boldsymbol{A}-\lambda\boldsymbol{I}]\{x\} = [0]$$

For $\lambda = 2$

$$\begin{bmatrix}5-2 & 3\\1 & 3-2\end{bmatrix}\begin{bmatrix}X_1\\X_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix}3 & 3\\1 & 1\end{bmatrix}\begin{bmatrix}X_1\\X_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$X_1+X_2 = 0 \qquad \Rightarrow X_1 = -X_2$$

So $$\text{eigen vector} = \begin{Bmatrix}1\\-1\end{Bmatrix}$$

$$\text{Magnitude of eigen vector} = \sqrt{(1)^2+(1)^2} = \sqrt{2}$$

$$\text{Normalized eigen vector} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$

SOL 1.8 Option (D) is correct.

Given : No. of Red balls = 4

No. of Black ball = 6

3 balls are selected randomly one after another, without replacement.

1 red and 2 black balls are will be selected as following

Manners	Probability for these sequence
$R\ B\ B$	$\frac{4}{10}\times\frac{6}{9}\times\frac{5}{8}=\frac{1}{6}$
$B\ R\ B$	$\frac{6}{10}\times\frac{4}{9}\times\frac{5}{8}=\frac{1}{6}$
$B\ B\ R$	$\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}=\frac{1}{6}$

Hence Total probability of selecting 1 red and 2 black ball is

$$P = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$$

SOL 1.9 Option (A) is correct.

We have $$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0 \qquad ...(1)$$

Let $x=e^z$ then $z = \log x$

$$\frac{dz}{dx}=\frac{1}{x}$$

So, we get $$\frac{dy}{dx}=\left(\frac{dy}{dz}\right)\left(\frac{dz}{dx}\right)=\frac{1}{x}\frac{dy}{dz}$$

$$x\frac{dy}{dx}=Dy \qquad \text{where } \frac{d}{dz}=D$$

Again $$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dz}\right)=\frac{-1}{x^2}\frac{dy}{dz}+\frac{1}{x}\frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx}$$

$$=\frac{-1}{x^2}\frac{dy}{dz}+\frac{1}{x}\frac{d^2y}{dz^2}\frac{dz}{dx}=\frac{1}{x^2}\left(\frac{d^2y}{dz^2}-\frac{dy}{dz}\right)$$

$$\frac{x^2d^2y}{dx^2}=(D^2-D)\,y=D(D-1)\,y$$

Now substitute in equation (i)

$$[D(D-1)+D-4]\,y=0$$

$$(D^2-4)\,y=0 \Rightarrow D=\pm 2$$

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So the required solution is $y = C_1 x^2 + C_2 x^{-2}$ ...(ii)

From the given limits $y(0) = 0$, equation (ii) gives

$$0 = C_1 \times 0 + C_2$$

$$C_2 = 0$$

And from $y(1) = 1$, equation (ii) gives

$$1 = C_1 + C_2$$

$$C_1 = 1$$

Substitute C_1 & C_2 in equation (ii), the required solution be

$$y = x^2$$

SOL 1.10 Option (C) is correct.

For given equation matrix form is as follows

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$$

The augmented matrix is

$$\left[\boldsymbol{A} : \boldsymbol{B}\right] = \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 2 & 1 & 2 & : & 5 \\ 1 & -1 & 1 & : & 1 \end{bmatrix} \qquad R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & -3 & 0 & : & -3 \\ 0 & -3 & 0 & : & -3 \end{bmatrix} \qquad R_3 \to R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & -3 & 0 & : & -3 \\ 0 & 0 & 0 & : & 0 \end{bmatrix} \qquad R_2 \to R_2/-3$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 4 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

This gives rank of $\boldsymbol{A}$, $\rho(A) = 2$ and Rank of $\left[\boldsymbol{A} : \boldsymbol{B}\right] = \rho\left[\boldsymbol{A} : \boldsymbol{B}\right] = 2$

Which is less than the number of unknowns (3)

$$\rho\left[\boldsymbol{A}\right] = \rho\left[\boldsymbol{A} : \boldsymbol{B}\right] = 2 < 3$$

Hence, this gives infinite No. of solutions.

SOL 1.11 Option (B) is correct.

$$\sin\theta = \theta - \frac{\theta^3}{\underline{|3}} + \frac{\theta^5}{\underline{|5}} - \frac{\theta^7}{\underline{|7}} + \dots\dots$$

SOL 1.12 Option (D) is correct.

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Let $$y = \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

$$= \lim_{\theta \to 0} \frac{\frac{d}{d\theta}(\sin\theta)}{\frac{d}{d\theta}(\theta)} = \lim_{\theta \to 0} \frac{\cos\theta}{1} \qquad \text{Applying L-Hospital rule}$$

$$= \frac{\cos 0}{1} = 1$$

SOL 1.13 Option (C) is correct

Let a square matrix

$$A = \begin{bmatrix} x & y \\ y & x \end{bmatrix}$$

We know that the characteristic equation for the eigen values is given by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} x-\lambda & y \\ y & x-\lambda \end{vmatrix} = 0$$

$$(x-\lambda)^2 - y^2 = 0$$

$$(x-\lambda)^2 = y^2$$

$$x - \lambda = \pm y \Rightarrow \lambda = x \pm y$$

So, eigen values are real if matrix is real and symmetric.

SOL 1.14 Option (A) is correct.

Let, $z_1 = (1+i)$, $z_2 = (2-5i)$

$$z = z_1 \times z_2 = (1+i)(2-5i)$$

$$= 2 - 5i + 2i - 5i^2 = 2 - 3i + 5 = 7 - 3i \qquad i^2 = -1$$

SOL 1.15 Option (D) is correct.

For a function, whose limits bounded between $-a$ to a and a is a positive real number. The solution is given by

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx\,; & f(x) \text{ is even} \\ 0 \qquad ; & f(x) \text{ is odd} \end{cases}$$

SOL 1.16 Option (C) is correct.

Let, $$f(x) = \int_1^3 \frac{1}{x}\,dx$$

From this function we get $a = 1$, $b = 3$ and $n = 3 - 1 = 2$

So, $$h = \frac{b-a}{n} = \frac{3-1}{2} = 1$$

We make the table from the given function $y = f(x) = \frac{1}{x}$ as follows :

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x	$f(x) = y = \frac{1}{x}$
$x = 1$	$y_1 = \frac{1}{1} = 1$
$x = 2$	$y_2 = \frac{1}{2} = 0.5$
$x = 3$	$y_3 = \frac{1}{3} = 0.333$

Applying the Simpson's $1/3^{rd}$ formula

$$\int_1^3 \frac{1}{x}\,dx = \frac{h}{3}[(y_1 + y_3) + 4y_2] = \frac{1}{3}[(1 + 0.333) + 4 \times 0.5]$$

$$= \frac{1}{3}[1.333 + 2] = \frac{3.333}{3} = 1.111$$

SOL 1.17 Option (D) is correct.

Given : $$\frac{dy}{dx} = (1 + y^2)\,x$$

$$\frac{dy}{(1 + y^2)} = xdx$$

Integrating both the sides, we get

$$\int \frac{dy}{1 + y^2} = \int xdx$$

$$\tan^{-1} y = \frac{x^2}{2} + c \quad \Rightarrow \quad y = \tan\left(\frac{x^2}{2} + c\right)$$

SOL 1.18 Option (D) is correct.

The probability of getting head $p = \frac{1}{2}$

And the probability of getting tail $q = 1 - \frac{1}{2} = \frac{1}{2}$

The probability of getting at least one head is

$$P(x \geq 1) = 1 - {}^5C_0(p)^5(q)^0 = 1 - 1 \times \left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$$

$$= 1 - \frac{1}{2^5} = \frac{31}{32}$$

SOL 1.19 Option (C) is correct.

Given system of equations are,

$$2x_1 + x_2 + x_3 = 0 \qquad \text{...(i)}$$

$$x_2 - x_3 = 0 \qquad \text{...(ii)}$$

$$x_1 + x_2 = 0 \qquad \text{...(iii)}$$

Adding the equation (i) and (ii) we have

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$$2x_1 + 2x_2 = 0$$

$$x_1 + x_2 = 0 \qquad \text{...(iv)}$$

We see that the equation (iii) and (iv) is same and they will meet at infinite points. Hence this system of equations have infinite number of solutions.

SOL 1.20 Option (D) is correct.

The volume of a solid generated by revolution about x-axis bounded by the function $f(x)$ and limits between a to b is given by

$$V = \int_a^b \pi y^2\, dx$$

Given $y = \sqrt{x}$ and $a = 1,\ b = 2$

Therefore, $$V = \int_1^2 \pi(\sqrt{x})^2\, dx = \pi \int_1^2 x dx = \pi\left[\frac{x^2}{2}\right]_1^2 = \pi\left[\frac{4}{2} - \frac{1}{2}\right] = \frac{3\pi}{2}$$

SOL 1.21 Option (B) is correct.

Given: $$\frac{d^3 f}{d\eta^3} + \frac{f}{2}\frac{d^2 f}{d\eta^2} = 0$$

Order is determined by the order of the highest derivation present in it. So, It is third order equation but it is a nonlinear equation because in linear equation, the product of f with $d^2 f/d\eta^2$ is not allow.

Therefore, it is a third order non-linear ordinary differential equation.

SOL 1.22 Option (D) is correct.

Let $$I = \int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$$

$$= [\tan^{-1} x]_{-\infty}^{\infty} = [\tan^{-1}(+\infty) - \tan^{-1}(-\infty)]$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi \qquad \tan^{-1}(-\theta) = -\tan^{-1}(\theta)$$

SOL 1.23 Option (B) is correct.

Let, $$z = \frac{3 + 4i}{1 - 2i}$$

Divide and multiply z by the conjugate of $(1 - 2i)$ to convert it in the form of $a + bi$ we have

$$z = \frac{3 + 4i}{1 - 2i} \times \frac{1 + 2i}{1 + 2i} = \frac{(3 + 4i)(1 + 2i)}{(1)^2 - (2i)^2}$$

$$= \frac{3 + 10i + 8i^2}{1 - 4i^2} = \frac{3 + 10i - 8}{1 - (-4)}$$

$$= \frac{-5 + 10i}{5} = -1 + 2i$$

$$|z| = \sqrt{(-1)^2 + (2)^2} = \sqrt{5} \qquad |a + ib| = \sqrt{a^2 + b^2}$$

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SOL 1.24 Option (C) is correct.

$$y = f(x) = \begin{cases} 2-3x & \text{if } x < \frac{2}{3} \\ 0 & \text{if } x = \frac{2}{3} \\ -(2-3x) & \text{if } x > \frac{2}{3} \end{cases}$$

Checking the continuity of the function.

At $x = \frac{2}{3}$, $Lf(x) = \lim_{h \to 0} f\left(\frac{2}{3} - h\right) = \lim_{h \to 0} 2 - 3\left(\frac{2}{3} - h\right)$

$$= \lim_{h \to 0} 2 - 2 + 3h = 0$$

and $Rf(x) = \lim_{h \to 0} f\left(\frac{2}{3} + h\right) = \lim_{h \to 0} 3\left(\frac{2}{3} + h\right) - 2$

$$= \lim_{h \to 0} 2 + 3h - 2 = 0$$

Since $L\lim_{h \to 0} f(x) = R\lim_{h \to 0} f(x)$

So, function is continuous $\forall\, x \in R$

Now checking the differentiability :

$$Lf'(x) = \lim_{h \to 0} \frac{f(\frac{2}{3} - h) - f(\frac{2}{3})}{-h} = \lim_{h \to 0} \frac{2 - 3(\frac{2}{3} - h) - 0}{-h}$$

$$= \lim_{h \to 0} \frac{2 - 2 + 3h}{-h} = \lim_{h \to 0} \frac{3h}{-h} = -3$$

and $Rf'(x) = \lim_{h \to 0} \frac{f(\frac{2}{3} + h) - f(\frac{2}{3})}{h}$

$$= \lim_{h \to 0} \frac{3(\frac{2}{3} + h) - 2 - 0}{h} = \lim_{h \to 0} \frac{2 + 3h - 2}{h} = 3$$

Since $Lf'\left(\frac{2}{3}\right) \neq Rf'\left(\frac{2}{3}\right)$, $f(x)$ is not differentiable at $x = \frac{2}{3}$.

SOL 1.25 Option (A) is correct.

Let, $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

And λ_1 and λ_2 are the eigen values of the matrix A.

The characteristic equation is written as

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0 \qquad \text{...(i)}$$

$$(2-\lambda)(3-\lambda) - 2 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0 \Rightarrow \lambda = 1 \;\&\; 4$$

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Putting $\lambda = 1$ in equation (i),

$$\begin{bmatrix} 2-1 & 2 \\ 1 & 3-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{where } \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ is eigen vector}$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 = 0 \text{ or } x_1 + 2x_2 = 0$$

Let $\quad x_2 = K$

Then $\quad x_1 + 2K = 0 \Rightarrow x_1 = -2K$

So, the eigen vector is

$$\begin{bmatrix} -2K \\ K \end{bmatrix} \text{ or } \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Since option A $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is in the same ratio of x_1 and x_2. Therefore option (A) is an eigen vector.

SOL 1.26 Option (A) is correct.

$f(t)$ is the inverse Laplace

So, $\quad f(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2(s+1)}\right]$

$$\frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

$$= \frac{As(1+s) + B(s+1) + Cs^2}{s^2(s+1)}$$

$$= \frac{s^2(A+C) + s(A+B) + B}{s^2(s+1)}$$

Compare the coefficients of s^2, s and constant terms and we get

$$A + C = 0; \; A + B = 0 \text{ and } B = 1$$

Solving above equation, we get $A = -1$, $B = 1$ and $C = 1$

Thus $\quad f(t) = \mathcal{L}^{-1}\left[-\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s+1}\right]$

$$= -1 + t + e^{-t} = t - 1 + e^{-t} \qquad \mathcal{L}^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$$

SOL 1.27 Option (C) is correct.

The box contains :

Number of washers = 2

Number of nuts = 3

Number of bolts = 4

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Total objects $= 2 + 3 + 4 = 9$

First two washers are drawn from the box which contain 9 items. So the probability of drawing 2 washers is,

$$P_1 = \frac{{}^2C_2}{{}^9C_2} == \frac{1}{\frac{9!}{7!2!}} = \frac{7!2!}{9 \times 8 \times 7!} = \frac{2}{9 \times 8} = \frac{1}{36} \qquad {}^nC_n = 1$$

After this box contains only 7 objects and then 3 nuts drawn from it. So the probability of drawing 3 nuts from the remaining objects is,

$$P_2 = \frac{{}^3C_3}{{}^7C_3} = \frac{1}{\frac{7!}{4!3!}} = \frac{4!3!}{7 \times 6 \times 5 \times 4!} = \frac{1}{35}$$

After this box contain only 4 objects, probability of drawing 4 bolts from the box,

$$P_3 = \frac{{}^4C_4}{{}^4C_4} = \frac{1}{1} = 1$$

Therefore the required probability is,

$$P = P_1P_2P_3 = \frac{1}{36} \times \frac{1}{35} \times 1 = \frac{1}{1260}$$

SOL 1.28 Option (B) is correct.

Given : $h = 60^\circ - 0 = 60^\circ$

$$h = 60 \times \frac{\pi}{180} = \frac{\pi}{3} = 1.047 \text{ radians}$$

From the table, we have

$y_0 = 0, \; y_1 = 1066, \; y_2 = -323, \; y_3 = 0, \; y_4 = 323, \; y_5 = -355$ and $y_6 = 0$

From the Simpson's 1/3rd rule the flywheel Energy is,

$$E = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

Substitute the values, we get

$$E = \frac{1.047}{3}[(0 + 0) + 4(1066 + 0 - 355) + 2(-323 + 323)]$$

$$= \frac{1.047}{3}[4 \times 711 + 2(0)] = 993 \text{ Nm rad (Joules/cycle)}$$

SOL 1.29 Option (A) is correct.

Given : $M = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$

And $[M]^T = [M]^{-1}$

We know that when $[A]^T = [A]^{-1}$ then it is called orthogonal matrix.

$$[M]^T = \frac{I}{[M]}$$

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$$[M]^T[M] = I$$

Substitute the values of M and M^T, we get

$$\begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \left(\frac{3}{5}\times\frac{3}{5}\right)+x^2 & \left(\frac{3}{5}\times\frac{4}{5}\right)+\frac{3}{5}x \\ \left(\frac{4}{5}\times\frac{3}{5}\right)+\frac{3}{5}x & \left(\frac{4}{5}\times\frac{4}{5}\right)+\left(\frac{3}{5}\times\frac{3}{5}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{9}{25}+x^2 & \frac{12}{25}+\frac{3}{5}x \\ \frac{12}{25}+\frac{3}{5}x & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing both sides a_{12} element,

$$\frac{12}{25}+\frac{3}{5}x = 0 \to x = -\frac{12}{25}\times\frac{5}{3} = -\frac{4}{5}$$

SOL 1.30 Option (C) is correct.

Let, $$\boldsymbol{V} = 3xz\boldsymbol{i} + 2xy\boldsymbol{j} - yz^2\boldsymbol{k}$$

We know divergence vector field of $\boldsymbol{V}$ is given by $(\nabla\cdot\boldsymbol{V})$

So, $$\nabla\cdot\boldsymbol{V} = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)\cdot(3xz\boldsymbol{i} + 2xy\boldsymbol{j} - yz^2\boldsymbol{k})$$

$$\nabla\cdot\boldsymbol{V} = 3z + 2x - 2yz$$

At point $P(1,1,1)$

$$(\nabla\cdot\boldsymbol{V})_{P(1,1,1)} = 3\times1 + 2\times1 - 2\times1\times1 = 3$$

SOL 1.31 Option (C) is correct.

Let $$f(s) = \mathcal{L}^{-1}\left[\frac{1}{s^2+s}\right]$$

First, take the function $\frac{1}{s^2+s}$ and break it by the partial fraction,

$$\frac{1}{s^2+s} = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{(s+1)} \qquad \left\{\begin{matrix}\text{Solve by} \\ \frac{1}{(s+1)} = \frac{A}{s} + \frac{B}{s+1}\end{matrix}\right\}$$

So, $$\mathcal{L}^{-1}\left(\frac{1}{s^2+s}\right) = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{(s+1)}\right] = \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] = 1 - e^{-t}$$

SOL 1.32 Option (D) is correct.

Total number of cases $= 2^3 = 8$

& Possible cases when coins are tossed simultaneously.

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H	H	H
H	H	T
H	T	H
T	H	H
H	T	T
T	H	T
T	T	H
T	T	T

From these cases we can see that out of total 8 cases 7 cases contain at least one head. So, the probability of come at least one head is $=\frac{7}{8}$

SOL 1.33 Option (C) is correct.

Given : $z = x+iy$ is a analytic function

$$f(z) = u(x,y)+iv(x,y)$$

$$u = xy \qquad ..(i)$$

Analytic function satisfies the Cauchy-Riemann equation.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

So from equation (i),

$$\frac{\partial u}{\partial x} = y \quad \Rightarrow \quad \frac{\partial v}{\partial y} = y$$

$$\frac{\partial u}{\partial y} = x \quad \Rightarrow \quad \frac{\partial v}{\partial x} = -x$$

Let $v(x,y)$ be the conjugate function of $u(x,y)$

$$dv = \frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy = (-x)\,dx+(y)\,dy$$

Integrating both the sides,

$$\int dv = -\int x dx+\int y dy$$

$$v = -\frac{x^2}{2}+\frac{y^2}{2}+k = \frac{1}{2}(y^2-x^2)+k$$

SOL 1.34 Option (A) is correct.

Given $\quad x\frac{dy}{dx}+y = x^4$

$$\frac{dy}{dx}+\left(\frac{1}{x}\right)y = x^3 \qquad ...(i)$$

It is a single order differential equation. Compare this with $\frac{dy}{dx}+Py = Q$ and we get

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$$P = \frac{1}{x} \qquad Q = x^3$$

Its solution will be

$$y(I.F.) = \int Q(I.F.)\,dx + C$$

$$I.F. = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log_e x} = x$$

Complete solution is given by,

$$yx = \int x^3 \times x dx + C = \int x^4\,dx + C = \frac{x^5}{5} + C \qquad \text{...(ii)}$$

and $y(1) = \frac{6}{5}$ at $x = 1 \Rightarrow y = \frac{6}{5}$ From equation (ii),

$$\frac{6}{5} \times 1 = \frac{1}{5} + C \;\Rightarrow\; C = \frac{6}{5} - \frac{1}{5} = 1$$

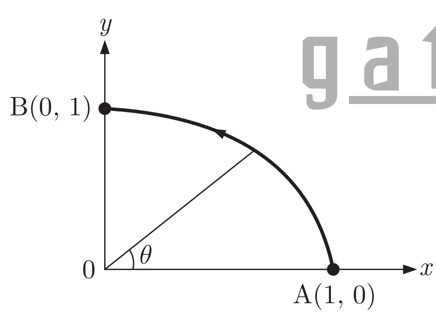
Then, from equation (ii), we get

$$yx = \frac{x^5}{5} + 1 \Rightarrow y = \frac{x^4}{5} + \frac{1}{x}$$

SOL 1.35 Option (B) is correct.

The equation of circle with unit radius and centre at origin is given by,

$$x^2 + y^2 = 1$$



Finding the integration of $(x + y)^2$ on path AB traversed in counter-clockwise sense So using the polar form

Let: $x = \cos\theta$, $y = \sin\theta$, and $r = 1$

So put the value of x and y and limits in first quadrant between 0 to $\pi/2$.

Hence,

$$I = \int_0^{\pi/2} (\cos\theta + \sin\theta)^2\,d\theta$$

$$= \int_0^{\pi/2} (\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta)\,d\theta$$

$$= \int_0^{\pi/2} (1 + \sin 2\theta)\,d\theta$$

Integrating above equation, we get

$$= \left[\theta - \frac{\cos 2\theta}{2}\right]_0^{\pi/2} = \left[\left(\frac{\pi}{2} - \frac{\cos\pi}{2}\right) - \left(0 - \frac{\cos 0}{2}\right)\right]$$

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$$= \left(\frac{\pi}{2} + \frac{1}{2}\right) - \left(-\frac{1}{2}\right) = \frac{\pi}{2} + 1$$

SOL 1.36 Option (A) is correct.

The given equation of surface is

$$z^2 = 1 + xy \qquad ...(i)$$

Let $P(x, y, z)$ be the nearest point on the surface (i), then distance from the origin is

$$d = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$

$$d^2 = x^2 + y^2 + z^2$$

$$z^2 = d^2 - x^2 - y^2 \qquad ...(ii)$$

From equation (i) and (ii), we get

$$d^2 - x^2 - y^2 = 1 + xy$$

$$d^2 = x^2 + y^2 + xy + 1$$

Let $\quad f(x, y) = d^2 = x^2 + y^2 + xy + 1 \qquad ...(iii)$

The $f(x, y)$ be the maximum or minimum according to d^2 maximum or minimum.

Differentiating equation (iii) w.r.t x and y respectively, we get

$$\frac{\partial f}{\partial x} = 2x + y \text{ or } \frac{\partial f}{\partial y} = 2y + x$$

Applying maxima minima principle and putting $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ equal to zero,

$$\frac{\partial f}{\partial x} = 2x + y = 0 \text{ or } \frac{\partial f}{\partial y} = 2y + x = 0$$

Solving these equations, we get $x = 0$, $y = 0$

So, $x = y = 0$ is only one stationary point.

Now $\quad p = \frac{\partial^2 f}{\partial x^2} = 2$

$$q = \frac{\partial^2 f}{\partial x \, \partial y} = 1$$

$$r = \frac{\partial^2 f}{\partial y^2} = 2$$

or $\quad pr - q^2 = 4 - 1 = 3 > 0$ and r is positive.

So, $\quad f(x, y) = d^2$ is minimum at (0,0).

Hence minimum value of d^2 at (0,0).

$$d^2 = x^2 + y^2 + xy + 1 = 1$$

$$d = 1 \text{ or } f(x, y) = 1$$

So, the nearest point is

$$z^2 = 1 + xy = 1 + 0$$

$\Rightarrow \quad z = \pm 1$

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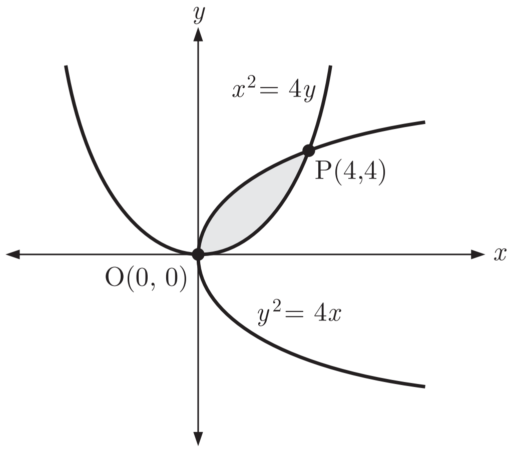
SOL 1.37 Option (A) is correct.

Given : $y^2 = 4x$ and $x^2 = 4y$ draw the curves from the given equations,



The shaded area shows the common area. Now finding the intersection points of the curves.

$$y^2 = 4x = 4\sqrt{4y} = 8\sqrt{y} \qquad x = \sqrt{4y} \text{ From second curve}$$

Squaring both sides

$$y^4 = 8 \times 8 \times y \Rightarrow y(y^3 - 64) = 0$$

$$y = 4 \ \& \ 0$$

Similarly put $y = 0$ in curve $x^2 = 4y$

$$x^2 = 4 \times 0 = 0 \Rightarrow x = 0$$

And Put $y = 4$

$$x^2 = 4 \times 4 = 16 \quad x = 4$$

So, $x = 4, 0$

Therefore the intersection points of the curves are (0,0) and (4,4).

So the enclosed area is given by

$$A = \int_{x_1}^{x_2} (y_1 - y_2)\, dx$$

Put y_1 and y_2 from the equation of curves $y^2 = 4x$ and $x^2 = 4y$

$$A = \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = 2\int_0^4 \sqrt{x}\, dx - \frac{1}{4}\int_0^4 x^2\, dx$$

Integrating the equation, we get

$$A = 2\left[\frac{2}{3}x^{3/2}\right]_0^4 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$$

$$= \frac{4}{3} \times 4^{3/2} - \frac{1}{4} \times \frac{4^3}{3} = \frac{4}{3} \times 8 - \frac{16}{3} = \frac{16}{3}$$

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SOL 1.38 Option (A) is correct.

The cumulative distribution function

$$f(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a < x < b \\ 0, & x \ge b \end{cases}$$

and density function

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x \le b \\ 0, & a > x, x > b \end{cases}$$

Mean $$E(x) = \sum_{x=a}^{b} x f(x) = \frac{a+b}{2}$$

$$\text{Variance} = x^2 f(x) - \overline{x}^2 = x^2 f(x) - [x f(x)]^2$$

Substitute the value of $f(x)$

$$\text{Variance} = \sum_{x=a}^{b} x^2 \frac{1}{b-a} dx - \left\{ \sum_{x=a}^{b} x \frac{1}{b-a} dx \right\}^2$$

$$= \left[\frac{x^3}{3(b-a)} \right]_a^b - \left[\left\{ \frac{x^2}{2(b-a)} \right\}_a^b \right]^2$$

$$= \frac{b^3 - a^3}{3(b-a)} - \frac{(b^2 - a^2)^2}{4(b-a)^2}$$

$$= \frac{(b-a)(b^2 + ab + a^2)}{3(b-a)} - \frac{(b+a)^2 (b-a)^2}{4(b-a)^2}$$

$$= \frac{4(b^2 + ab + a^2) + 3(a+b)^2}{12} = \frac{(b-a)^2}{12}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \sqrt{\frac{(b-a)^2}{12}} = \frac{(b-a)}{\sqrt{12}}$$

Given : $b = 1$, $a = 0$

So, standard deviation $= \dfrac{1-0}{\sqrt{12}} = \dfrac{1}{\sqrt{12}}$

SOL 1.39 Option (C) is correct.

Taylor's series expansion of $f(x)$ is given by,

$$f(x) = f(a) + \frac{(x-a)}{\lfloor 1} f'(a) + \frac{(x-a)^2}{\lfloor 2} f''(a) + \frac{(x-a)^3}{\lfloor 3} f'''(a) + \dots$$

Then from this expansion the coefficient of $(x-a)^4$ is $\dfrac{f''''(a)}{\lfloor 4}$

Given $a = 2$

$$f(x) = e^x$$

$$f'(x) = e^x$$

$$f''(x) = e^x$$

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$$f'''(x) = e^x$$
$$f''''(x) = e^x$$

Hence, for $a = 2$ the coefficient of $(x - a)^4$ is $\frac{e^2}{\lfloor 4}$

SOL 1.40 Option (D) is correct.

Given : $\ddot{x} + 3x = 0$ and $x(0) = 1$

$$(D^2 + 3)x = 0 \qquad D = \frac{d}{dt}$$

The auxiliary Equation is written as

$$m^2 + 3 = 0$$
$$m = \pm\sqrt{3}\,i = 0 \pm \sqrt{3}\,i$$

Here the roots are imaginary

$$m_1 = 0 \text{ and } m_2 = \sqrt{3}$$

Solution is given by

$$x = e^{m_1 t}(A\cos m_2 t + B\sin m_2 t)$$
$$= e^0[A\cos\sqrt{3}\,t + B\sin\sqrt{3}\,t]$$
$$= [A\cos\sqrt{3}\,t + B\sin\sqrt{3}\,t] \qquad \text{...(i)}$$

Given : $x(0) = 1$ at $t = 0$, $x = 1$

Substituting in equation (i),

$$1 = [A\cos\sqrt{3}(0) + B\sin\sqrt{3}(0)] = A + 0$$
$$A = 1$$

Differentiateing equation (i) w.r.t. t,

$$\dot{x} = \sqrt{3}[-A\sin\sqrt{3}\,t + B\cos\sqrt{3}\,t] \qquad \text{...(ii)}$$

Given $\dot{x}(0) = 0$ at $t = 0$, $\dot{x} = 0$

Substituting in equation (ii), we get

$$0 = \sqrt{3}[-A\sin 0 + B\cos 0]$$
$$B = 0$$

Substituting A & B in equation (i)

$$x = \cos\sqrt{3}\,t$$
$$x(1) = \cos\sqrt{3} = 0.99$$

SOL 1.41 Option (B) is correct.

Let $$f(x) = \lim_{x \to 8} \frac{x^{1/3} - 2}{(x - 8)} \qquad \frac{0}{0} \text{ form}$$
$$= \lim_{x \to 8} \frac{\frac{1}{3}x^{-2/3}}{1} \qquad \text{Applying L-Hospital rule}$$

Substitute the limits, we get

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$$f(x) = \frac{1}{3}(8)^{-2/3} = \frac{1}{3}(2^3)^{-2/3} = \frac{1}{4\times 3} = \frac{1}{12}$$

SOL 1.42 Option (A) is correct.

In a coin probability of getting Head

$$p = \frac{1}{2} = \frac{\text{No. of Possible cases}}{\text{No. of Total cases}}$$

Probability of getting tail

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

So the probability of getting Heads exactly three times, when coin is tossed 4 times is

$$P = {}^4C_3(p)^3(q)^1 = {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1$$

$$= 4\times\frac{1}{8}\times\frac{1}{2} = \frac{1}{4}$$

SOL 1.43 Option (C) is correct.

Let,
$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$$

Let the eigen values of this matrix are $\lambda_1, \lambda_2 \,\&\, \lambda_3$

Here one values is given so let $\lambda_1 = 3$

We know that

Sum of eigen values of matrix= Sum of the diagonal element of matrix A

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 + 0 + p$$

$$\lambda_2 + \lambda_3 = 1 + p - \lambda_1 = 1 + p - 3 = p - 2$$

SOL 1.44 Option (D) is correct.

We know that the divergence is defined as $\nabla\cdot\boldsymbol{V}$

Let
$$\boldsymbol{V} = (x-y)\,\boldsymbol{i} + (y-x)\,\boldsymbol{j} + (x+y+z)\,\boldsymbol{k}$$

And
$$\nabla = \left(\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right)$$

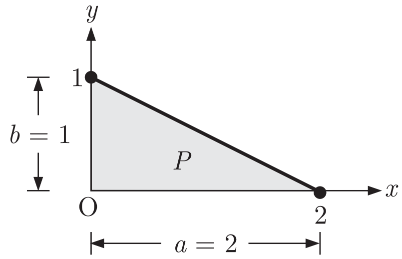
So,
$$\nabla\cdot\boldsymbol{V} = \left(\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right)\cdot\left[(x-y)\,\boldsymbol{i} + (y-x)\,\boldsymbol{j} + (x+y+z)\,\boldsymbol{k}\right]$$

$$= \frac{\partial}{\partial x}(x-y) + \frac{\partial}{\partial y}(y-x) + \frac{\partial}{\partial z}(x+y+z)$$

$$= 1 + 1 + 1 = 3$$

SOL 1.45 Option (A) is correct.

Given :



The equation of line in intercept form is given by

$$\frac{x}{2}+\frac{y}{1}=1 \qquad\qquad \frac{x}{a}+\frac{y}{b}=1$$

$$x+2y=2 \Rightarrow x=2(1-y)$$

The limit of x is between 0 to $x=2(1-y)$ and y is 0 to 1,

Now $$\iint_p xydxdy=\int_{y=0}^{y=1}\int_{x=0}^{2(1-y)} xydxdy=\int_{y=0}^{y=1}\left[\frac{x^2}{2}\right]_0^{2(1-y)} ydy$$

$$=\int_{y=0}^{y=1} y\left[\frac{4(1-y)^2}{2}-0\right]dy$$

$$=\int_{y=0}^{y=1} 2y(1+y^2-2y)\,dy=\int_{y=0}^{y=1} 2(y+y^3-2y^2)\,dy$$

Again Integrating and substituting the limits, we get

$$\iint_p xydxdy=2\left[\frac{y^2}{2}+\frac{y^4}{4}-\frac{2y^3}{3}\right]_0^1=2\left[\frac{1}{2}+\frac{1}{4}-\frac{2}{3}-0\right]$$

$$=2\left[\frac{6+3-8}{12}\right]=\frac{2}{12}=\frac{1}{6}$$

SOL 1.46 Option (B) is correct.

Direction derivative of a function f along a vector $\boldsymbol{P}$ is given by

$$\boldsymbol{a}=\text{grad } f\cdot\frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$

where $$\text{grad } f=\left(\frac{\partial f}{\partial x}\boldsymbol{i}+\frac{\partial f}{\partial y}\boldsymbol{j}+\frac{\partial f}{\partial z}\boldsymbol{k}\right)$$

$$f(x,y,z)=x^2+2y^2+z,\quad \boldsymbol{a}=3\boldsymbol{i}-4\boldsymbol{j}$$

$$\boldsymbol{a}=\text{grad}\,(x^2+2y^2+z)\cdot\frac{3\boldsymbol{i}-4\boldsymbol{j}}{\sqrt{(3)^2+(-4)^2}}$$

$$=(2x\boldsymbol{i}+4y\boldsymbol{j}+\boldsymbol{k})\cdot\frac{(3\boldsymbol{i}-4\boldsymbol{j})}{\sqrt{25}}=\frac{6x-16y}{5}$$

At point $P(1,1,2)$ the direction derivative is

$$\boldsymbol{a}=\frac{6\times1-16\times1}{5}=-\frac{10}{5}=-2$$

SOL 1.47 Option (B) is correct.

Given : $2x+3y=4$

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$$x + y + z = 4$$
$$x + 2y - z = a$$

It is a set of non-homogenous equation, so the augmented matrix of this system is

$$[A:B] = \begin{bmatrix} 2 & 3 & 0 & : & 4 \\ 1 & 1 & 1 & : & 4 \\ 1 & 2 & -1 & : & a \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 3 & 0 & : & 4 \\ 0 & -1 & 2 & : & 4 \\ 2 & 3 & 0 & : & 4+a \end{bmatrix} \qquad R_3 \to R_3 + R_2,\ R_2 \to 2R_2 - R_1$$

$$\sim \begin{bmatrix} 2 & 3 & 0 & : & 4 \\ 0 & -1 & 2 & : & 4 \\ 0 & 0 & 0 & : & a \end{bmatrix} \qquad R_3 \to R_3 - R_1$$

So, for a unique solution of the system of equations, it must have the condition

$$\rho[A:B] = \rho[A]$$

So, when putting $a = 0$

We get $\rho[A:B] = \rho[A]$

SOL 1.48 Option (D) is correct.

Here we check all the four options for unbounded condition.

(A) $$\int_0^{\pi/4} \tan x dx = \left[\log|\sec x|\right]_0^{\pi/4} = \left[\log\left|\sec\frac{\pi}{4}\right| - \log|\sec 0|\right]$$
$$= \log\sqrt{2} - \log 1 = \log\sqrt{2}$$

(B) $$\int_0^{\infty} \frac{1}{x^2+1} dx = \left[\tan^{-1}x\right]_0^{\infty} = \tan^{-1}\infty - \tan^{-1}(0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

(C) $$\int_0^{\infty} xe^{-x} dx$$

Let $$I = \int_0^{\infty} xe^{-x} dx = x\int_0^{\infty} e^{-x} dx - \int_0^{\infty}\left[\frac{d}{dx}(x)\int e^{-x} dx\right]dx$$

$$= [-xe^{-x}]_0^{\infty} + \int_0^{\infty} e^{-x} dx = [-xe^{-x} - e^{-x}]_0^{\infty} = [-e^{-x}(x+1)]_0^{\infty}$$
$$= -[0-1] = 1$$

(D) $$\int_0^1 \frac{1}{1-x} dx = -\int_0^1 \frac{1}{x-1} dx = -[\log(x-1)]_0^1 - [\log 0 - \log(-1)]$$

Both log 0 and log (–1) undefined so it is unbounded.

SOL 1.49 Option (A) is correct.

Let $$I = \oint f(z)\, dz \text{ and } f(z) = \frac{\cos z}{z}$$

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Then $$I = \oint \frac{\cos z}{z} dz = \oint \frac{\cos z}{\lvert z-0 \rvert} dz \quad \text{...(i)}$$

Given that $\lvert z \rvert = 1$ for unit circle. From the Cauchy Integral formula

$$\oint \frac{f(z)}{z-a} dz = 2\pi i f(a) \quad \text{...(ii)}$$

Compare equation (i) and (ii), we can say that,

$$a = 0 \text{ and } f(z) = \cos z$$

Or, $$f(a) = f(0) = \cos 0 = 1$$

Now from equation (ii) we get

$$\oint \frac{f(z)}{z-0} dz = 2\pi i \times 1 = 2\pi i \qquad a = 0$$

SOL 1.50 Option (D) is correct.

Given $$y = \frac{2}{3} x^{3/2} \quad \text{...(i)}$$

We know that the length of curve is given by $\int_{x_1}^{x_2} \left\{ \sqrt{\left(\frac{dy}{dx}\right)^2 + 1} \right\} dx$ ...(ii)

Differentiate equation(i) w.r.t. x

$$\frac{dy}{dx} = \frac{2}{3} \times \frac{3}{2} x^{\frac{3}{2}-1} = x^{1/2} = \sqrt{x}$$

Substitute the limit $x_1 = 0$ to $x_2 = 1$ and $\frac{dy}{dx}$ in equation (ii), we get

$$\mathcal{L} = \int_0^1 \left(\sqrt{(\sqrt{x})^2 + 1}\right) dx = \int_0^1 \sqrt{x+1}\, dx$$

$$= \left[\frac{2}{3}(x+1)^{3/2}\right]_0^1 = 1.22$$

SOL 1.51 Option (B) is correct.

Let $$A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$$ λ_1 and λ_2 is the eigen values of the matrix.

For eigen values characteristic matrix is,

$$\lvert A - \lambda I \rvert = 0$$

$$\left\lvert \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\rvert = 0$$

$$\begin{vmatrix} (1-\lambda) & 2 \\ 0 & (2-\lambda) \end{vmatrix} = 0 \quad \text{...(i)}$$

$$(1-\lambda)(2-\lambda) = 0 \Rightarrow \lambda = 1 \,\&\, 2$$

So, Eigen vector corresponding to the $\lambda = 1$ is,

$$\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a \end{bmatrix} = 0$$

$$2a + a = 0 \Rightarrow a = 0$$

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Again for $\lambda = 2$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ b \end{bmatrix} = 0$$

$$-1 + 2b = 0 \qquad b = \frac{1}{2}$$

Then sum of $\quad a \,\&\, b \Rightarrow a + b = 0 + \frac{1}{2} = \frac{1}{2}$

SOL 1.52 Option (C) is correct.

Given $\qquad f(x, y) = y^x$

First partially differentiate the function w.r.t. y

$$\frac{\partial f}{\partial y} = xy^{x-1}$$

Again differentiate. it w.r.t. x

$$\frac{\partial^2 f}{\partial x \partial y} = y^{x-1}(1) + x(y^{x-1}\log y) = y^{x-1}(x\log y + 1)$$

At : $\qquad x = 2,\ y = 1$

$$\frac{\partial^2 f}{\partial x \partial y} = (1)^{2-1}(2\log 1 + 1) = 1(2 \times 0 + 1) = 1$$

SOL 1.53 Option (A) is correct.

Given : $\qquad y'' + 2y' + y = 0$

$$(D^2 + 2D + 1)\,y = 0 \qquad \text{where } D = d/dx$$

The auxiliary equation is

$$m^2 + 2m + 1 = 0$$

$$(m + 1)^2 = 0,\ m = -1, -1$$

The roots of auxiliary equation are equal and hence the general solution of the given differential equation is,

$$y = (C_1 + C_2 x)\, e^{m_1 x} = (C_1 + C_2 x)\, e^{-x} \qquad ..(i)$$

Given $y(0) = 0$ at $x = 0$, $\quad \Rightarrow y = 0$

Substitute in equation (i), we get

$$0 = (C_1 + C_2 \times 0)\, e^{-0}$$

$$0 = C_1 \times 1 \Rightarrow C_1 = 0$$

Again $y(1) = 0$, at $x = 1 \Rightarrow y = 0$

Substitute in equation (i), we get

$$0 = [C_1 + C_2 \times (1)]\, e^{-1} = [C_1 + C_2]\frac{1}{e}$$

$$C_1 + C_2 = 0 \quad \Rightarrow C_2 = 0$$

Substitute C_1 and C_2 in equation (i), we get

$$y = (0 + 0x)\, e^{-x} = 0$$

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And $y(0.5) = 0$

SOL 1.54 Option (B) is correct.

Given : $y = x^2$ ...(i)

and interval [1, 5]

At $x = 1 \Rightarrow y = 1$

And at $x = 5 \quad y = (5)^2 = 25$

Here the interval is bounded between 1 and 5

So, the minimum value at this interval is 1.

SOL 1.55 Option (A) is correct

Let square matrix

$$A = \begin{bmatrix} x & y \\ y & x \end{bmatrix}$$

The characteristic equation for the eigen values is given by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} x-\lambda & y \\ y & x-\lambda \end{vmatrix} = 0$$

$$(x-\lambda)^2 - y^2 = 0$$

$$(x-\lambda)^2 = y^2$$

$$x - \lambda = \pm y$$

$$\lambda = x \pm y$$

So, eigen values are real if matrix is real and symmetric.

SOL 1.56 Option (B) is correct.

The Cauchy-Reimann equation, the necessary condition for a function $f(z)$ to be analytic is

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\frac{\partial \varphi}{\partial y} = -\frac{\partial \psi}{\partial x}$$ when $\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial x}$ exist.

SOL 1.57 Option (A) is correct.

Given : $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial y} = 0$

Order is determined by the order of the highest derivative present in it.

Degree is determined by the degree of the highest order derivative present in it after the differential equation is cleared of radicals and fractions.

So, degree = 1 and order = 2

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SOL 1.58 Option (B) is correct.

Given $$y = x + \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}} \qquad ...(i)$$

$$y - x = \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$$

Squaring both the sides,

$$(y-x)^2 = x + \sqrt{x + \sqrt{x + ...\infty}}$$

$$(y-x)^2 = y \qquad \text{From equation (i)}$$

$$y^2 + x^2 - 2xy = y \qquad ...(ii)$$

We have to find $y(2)$, put $x = 2$ in equation (ii),

$$y^2 + 4 - 4y = y$$

$$y^2 - 5y + 4 = 0$$

$$(y-4)(y-1) = 0$$

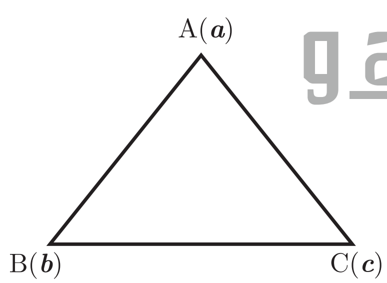
$$y = 1, 4$$

From Equation (i) we see that

For $y(2)$ $$y = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + ...\infty}}} > 2$$

Therefore, $$y = 4$$

SOL 1.59 Option (B) is correct.



Vector area of ΔABC,

$$A = \frac{1}{2}\boldsymbol{BC} \times \boldsymbol{BA} = \frac{1}{2}(\boldsymbol{c} - \boldsymbol{b}) \times (\boldsymbol{a} - \boldsymbol{b})$$

$$= \frac{1}{2}[\boldsymbol{c} \times \boldsymbol{a} - \boldsymbol{c} \times \boldsymbol{b} - \boldsymbol{b} \times \boldsymbol{a} + \boldsymbol{b} \times \boldsymbol{b}]$$

$$= \frac{1}{2}[\boldsymbol{c} \times \boldsymbol{a} + \boldsymbol{b} \times \boldsymbol{c} + \boldsymbol{a} \times \boldsymbol{b}]$$

$$\boldsymbol{b} \times \boldsymbol{b} = 0 \text{ and } \boldsymbol{c} \times \boldsymbol{b} = -(\boldsymbol{b} \times \boldsymbol{c})$$

$$= \frac{1}{2}[(\boldsymbol{a} - \boldsymbol{b}) \times (\boldsymbol{a} - \boldsymbol{c})]$$

SOL 1.60 Option (C) is correct.

Given : $$\frac{dy}{dx} = y^2 \text{ or } \frac{dy}{y^2} = dx$$

Integrating both the sides

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$$\int \frac{dy}{y^2} = \int dx$$

$$-\frac{1}{y} = x + C \qquad \text{...(i)}$$

Given $y(0) = 1$ at $x = 0 \Rightarrow y = 1$

Put in equation (i) for the value of C

$$-\frac{1}{1} = 0 + C \Rightarrow C = -1$$

From equation (i),

$$-\frac{1}{y} = x - 1$$

$$y = -\frac{1}{x-1}$$

For this value of y, $x - 1 \neq 0$ or $x \neq 1$

And $x < 1$ or $x > 1$

SOL 1.61 Option (A) is correct.

Let $\phi(t) = \int_0^t f(t)\, dt$ and $\phi(0) = 0$ then $\phi'(t) = f(t)$

We know the formula of Laplace transforms of $\phi'(t)$ is

$$L[\phi'(t)] = sL[\phi(t)] - \phi(0) = sL[\phi(t)] \qquad \phi(0) = 0$$

$$L[\phi(t)] = \frac{1}{s} L[\phi'(t)]$$

Substitute the values of $\phi(t)$ and $\phi'(t)$, we get

$$L\left[\int_0^t f(t)\, dt\right] = \frac{1}{s} L[f(t)]$$

or $$L\left[\int_0^t f(t)\, dt\right] = \frac{1}{s} F(s)$$

SOL 1.62 Option (A) is correct.

From the Trapezoidal Method

$$\int_a^b f(x)\, dx = \frac{h}{2}[f(x_0) + 2f(x_1) + 2f(x_2) \,.....2f(x_{n-1}) + f(x_n)] \qquad \text{...(i)}$$

$$\text{Interval } h = \frac{2\pi - 0}{8} = \frac{\pi}{4}$$

Find $\int_0^{2\pi} \sin x dx$ Here $f(x) = \sin x$

Table for the interval of $\pi/4$ is as follows

Angle θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$f(x) = \sin x$	0	0.707	1	0.707	0	-0.707	-1	-0.707	0

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Now from equation(i),

$$\int_0^{2\pi} \sin x dx = \frac{\pi}{8}[0 + 2(0.707 + 1 + 0.707 + 0 - 0.707 - 1 - 0.0707 + 0)]$$

$$= \frac{\pi}{8} \times 0 = 0$$

SOL 1.63 Option (D) is correct.

The X and Y be two independent random variables.

So, $$E(XY) = E(X)E(Y) \quad \text{(i)}$$

& covariance is defined as

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$= E(X)E(Y) - E(X)E(Y) \quad \text{From eqn. (i)}$$

$$= 0$$

For two independent random variables

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$$

and $$E(X^2 Y^2) = E(X^2)E(Y^2)$$

So, option (D) is incorrect.

SOL 1.64 Option (B) is correct.

Let, $$f(x) = \lim_{x \to 0} \frac{e^x - \left(1 + x + \frac{x^2}{2}\right)}{x^3} \quad \frac{0}{0} \text{ form}$$

$$= \lim_{x \to 0} \frac{e^x - (1 + x)}{3x^2} \quad \frac{0}{0} \text{ form}$$

$$= \lim_{x \to 0} \frac{e^x - 1}{6x} \quad \frac{0}{0} \text{ form}$$

$$= \lim_{x \to 0} \frac{e^x}{6} = \frac{e^0}{6} = \frac{1}{6}$$

SOL 1.65 Option (B) is correct.

Let, $$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

Let λ is the eigen value of the given matrix then characteristic matrix is

$$|A - \lambda I| = 0 \qquad \text{Here } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \text{Identity matrix}$$

$$\begin{vmatrix} 2 - \lambda & 1 \\ 0 & 2 - \lambda \end{vmatrix} = 0$$

$$(2 - \lambda)^2 = 0$$

$$\lambda = 2, 2$$

So, only one eigen vector.

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SOL 1.66 Option (D) is correct.

	Column I		
P.	Gauss-Seidel method	**4.**	Linear algebraic equation
Q.	Forward Newton-Gauss method	**1.**	Interpolation
R.	Runge-Kutta method	**2.**	Non-linear differential equation
S.	Trapezoidal Rule	**3.**	Numerical integration

So, correct pairs are, P-4, Q-1, R-2, S-3

SOL 1.67 Option (B) is correct.

Given : $\frac{dy}{dx} + 2xy = e^{-x^2}$ and $y(0) = 1$

It is the first order linear differential equation so its solution is

$$y(I.F.) = \int Q(I.F.)\,dx + C$$

compare with $\frac{dy}{dx} + P(y) = Q$

So, $$I.F. = e^{\int P dx} = e^{\int 2x dx}$$

$$= e^{2\int x dx} = e^{2\times\frac{x^2}{2}} = e^{x^2}$$

The complete solution is,

$$ye^{x^2} = \int e^{-x^2} \times e^{x^2}\,dx + C$$

$$= \int dx + C = x + C$$

$$y = \frac{x+c}{e^{x^2}} \qquad \text{...(i)}$$

Given $y(0) = 1$

At $x = 0 \Rightarrow y = 1$

Substitute in equation (i), we get

$$1 = \frac{C}{1} \Rightarrow C = 1$$

Then $$y = \frac{x+1}{e^{x^2}} = (x+1)\,e^{-x^2}$$

SOL 1.68 Option (C) is correct.

The incorrect statement is, $S = \{x : x \in A \text{ and } x \in B\}$ represents the union of set A and set B.

The above symbol $(\in)$ denotes intersection of set A and set B. Therefore this statement is incorrect.

SOL 1.69 Option (D) is correct.

Total number of items = 100

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Number of defective items = 20
Number of Non-defective items = 80
Then the probability that both items are defective, when 2 items are selected at random is,

$$P = \frac{{}^{20}C_2\,{}^{80}C_0}{{}^{100}C_2} = \frac{\frac{20!}{18!2!}}{\frac{100!}{98!2!}} = \frac{\frac{20\times 19}{2}}{\frac{100\times 99}{2}} = \frac{19}{495}$$

Alternate Method :
Here two items are selected without replacement.
Probability of first item being defective is

$$P_1 = \frac{20}{100} = \frac{1}{5}$$

After drawing one defective item from box, there are 19 defective items in the 99 remaining items.
Probability that second item is defective,

$$P_2 = \frac{19}{899}$$

then probability that both are defective

$$P = P_1 \times P_2 = \frac{1}{5} \times \frac{19}{99} = \frac{19}{495}$$

SOL 1.70 Option (A) is correct.

Given : $$S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

Eigen values of this matrix is 5 and 1. We can say $\lambda_1 = 1 \quad \lambda_2 = 5$
Then the eigen value of the matrix

$$S^2 = S\,S \text{ is } \lambda_1^2,\ \lambda_2^2$$

Because. if $\lambda_1, \lambda_2, \lambda_3 ....$ are the eigen values of A, then eigen value of A^m are $\lambda_1^m, \lambda_2^m, \lambda_3^m ....$
Hence matrix S^2 has eigen values $(1)^2$ and $(5)^2 \Rightarrow 1$ and 25

SOL 1.71 Option (B) is correct.

Given $$f(x) = (x-8)^{2/3} + 1$$

The equation of line normal to the function is

$$(y - y_1) = m_2(x - x_1) \qquad ...(i)$$

Slope of tangent at point (0, 5) is

$$m_1 = f'(x) = \left[\frac{2}{3}(x-8)^{-1/3}\right]_{(0,5)}$$

$$m_1 = f'(x) = \frac{2}{3}(-8)^{-1/3} = -\frac{2}{3}(2^3)^{-\frac{1}{3}} = -\frac{1}{3}$$

We know the slope of two perpendicular curves is -1.

$$m_1 m_2 = -1$$

$$m_2 = -\frac{1}{m_1} = \frac{-1}{-1/3} = 3$$

The equation of line, from equation (i) is

$$(y-5) = 3(x-0)$$

$$y = 3x+5$$

SOL 1.72 Option (A) is correct.

Let
$$f(x) = \int_0^{\pi/3} e^{it}\,dt = \left[\frac{e^{it}}{i}\right]_0^{\pi/3} \Rightarrow \frac{e^{i\pi/3}}{i} - \frac{e^0}{i}$$

$$= \frac{1}{i}\left[e^{\frac{\pi}{3}i} - 1\right] = \frac{1}{i}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} - 1\right]$$

$$= \frac{1}{i}\left[\frac{1}{2} + i\frac{\sqrt{3}}{2} - 1\right] = \frac{1}{i}\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$$

$$= \frac{1}{i} \times \frac{i}{i}\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right] = -i\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right] \qquad i^2 = -1$$

$$= i\left[\frac{1}{2} - \frac{\sqrt{3}}{2}i\right] = \frac{1}{2}i - \frac{\sqrt{3}}{2}i^2 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

SOL 1.73 Option (B) is correct.

Given
$$f(x) = \frac{2x^2 - 7x + 3}{5x^2 - 12x - 9}$$

Then
$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{2x^2 - 7x + 3}{5x^2 - 12x - 9}$$

$$= \lim_{x \to 3} \frac{4x - 7}{10x - 12} \qquad \text{Applying L – Hospital rule}$$

Substitute the limit, we get

$$\lim_{x \to 3} f(x) = \frac{4 \times 3 - 7}{10 \times 3 - 12} = \frac{12 - 7}{30 - 12} = \frac{5}{18}$$

SOL 1.74 Option (A) is correct.

(P) Singular Matrix → Determinant is zero $|A| = 0$

(Q) Non-square matrix → An $m \times n$ matrix for which $m \neq n$, is called non-square matrix. Its determinant is not defined

(R) Real Symmetric Matrix → Eigen values are always real.

(S) Orthogonal Matrix → A square matrix A is said to be orthogonal if $AA^T = I$

Its determinant is always one.

SOL 1.75 Option (B) is correct.

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Given : $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 3e^{2x}$

$$[D^2 + 4D + 3]\, y = 3e^{2x} \qquad \frac{d}{dx} = D$$

The auxiliary Equation is,

$$m^2 + 4m + 3 = 0 \Rightarrow m = -1, -3$$

Then $C.F. = C_1 e^{-x} + C_2 e^{-3x}$

$$P.I. = \frac{3e^{2x}}{D^2 + 4D + 3} = \frac{3e^{2x}}{(D+1)(D+3)} \qquad \text{Put } D = 2$$

$$= \frac{3e^{2x}}{(2+1)(2+3)} = \frac{3e^{2x}}{3 \times 5} = \frac{e^{2x}}{5}$$

SOL 1.76 Option (C) is correct.

Given $EF = G$ where $G = I =$ Identity matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We know that the multiplication of a matrix and its inverse be a identity matrix

$$AA^{-1} = I$$

So, we can say that F is the inverse matrix of E

$$F = E^{-1} = \frac{[\text{adj}.E]}{|E|}$$

$$\text{adj}E = \begin{bmatrix} \cos\theta & -(\sin\theta) & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|E| = [\cos\theta \times (\cos\theta - 0)] - [(-\sin\theta) \times (\sin\theta - 0)] + 0$$

$$= \cos^2\theta + \sin^2\theta = 1$$

Hence, $F = \frac{[\text{adj}.E]}{|E|} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

SOL 1.77 Option (B) is correct.

The probability density function is,

$$f(t) = \begin{cases} 1 + t & \text{for} -1 \le t \le 0 \\ 1 - t & \text{for } 0 \le t \le 1 \end{cases}$$

For standard deviation first we have to find the mean and variance of the function.

$$\text{Mean } (\bar{t}) = \int_{-1}^{\infty} t f(t)\, dt = \int_{-1}^{0} t(1+t)\, dt + \int_{0}^{1} t(1-t)\, dt$$

$$= \int_{-1}^{0} (t + t^2)\, dt + \int_{0}^{1} (t - t^2)\, dt$$

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$$=\left[\frac{t^2}{2}+\frac{t^3}{3}\right]_{-1}^{0}+\left[\frac{t^2}{2}-\frac{t^3}{3}\right]_{0}^{1}=\left[-\frac{1}{2}+\frac{1}{3}\right]+\left[\frac{1}{2}-\frac{1}{3}\right]=0$$

And variance $(\sigma^2)=\int_{-\infty}^{\infty}(t-\bar{t})^2 f(t)\,dt$ $\qquad \bar{t}=0$

$$=\int_{-1}^{0}t^2(1+t)\,dt+\int_{0}^{1}t^2(1-t)\,dt$$

$$=\int_{-1}^{0}(t^2+t^3)\,dt+\int_{0}^{1}(t^2-t^3)\,dt$$

$$=\left[\frac{t^3}{3}+\frac{t^4}{4}\right]_{-1}^{0}+\left[\frac{t^3}{3}-\frac{t^4}{4}\right]_{0}^{1}$$

$$=-\left[-\frac{1}{3}+\frac{1}{4}\right]+\left[\frac{1}{3}-\frac{1}{4}-0\right]=\frac{1}{12}+\frac{1}{12}=\frac{1}{6}$$

Now, standard deviation

$$\sqrt{(\sigma^2)}\,s=\sqrt{\frac{1}{6}}=\frac{1}{\sqrt{6}}$$

SOL 1.78 Option (A) is correct.

The Stokes theorem is,

$$\oint_C \boldsymbol{F}\cdot dr=\iint_S(\nabla\times\boldsymbol{F})\cdot\boldsymbol{n}dS=\iint_S(\text{Curl }\boldsymbol{F})\cdot dS$$

Here we can see that the line integral $\oint_C \boldsymbol{F}\cdot dr$ and surface integral $\iint_S(\text{Curl }\boldsymbol{F})\cdot ds$ is related to the stokes theorem.

SOL 1.79 Option (B) is correct.

Let, P = defective items

Q = non-defective items

10% items are defective, then probability of defective items

$$P=0.1$$

Probability of non-defective item

$$Q=1-0.1=0.9$$

The Probability that exactly 2 of the chosen items are defective is

$$={}^{10}C_2(P)^2(Q)^8=\frac{10!}{8!2!}(0.1)^2(0.9)^8$$

$$=45\times(0.1)^2\times(0.9)^8=0.1937$$

SOL 1.80 Option (A) is correct.

Let $\qquad f(x)=\int_{-a}^{a}(\sin^6x+\sin^7x)\,dx$

$$=\int_{-a}^{a}\sin^6xdx+\int_{-a}^{a}\sin^7xdx$$

We know that

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$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0 & \text{when } f(-x) = -f(x); \text{odd function} \\ 2\int_0^a f(x) & \text{when } f(-x) = f(x); \text{even function} \end{cases}$$

Now, here $\sin^6 x$ is an even function and $\sin^7 x$ is an odd function. Then,

$$f(x) = 2\int_0^a \sin^6 x dx + 0 = 2\int_0^a \sin^6 x dx$$

SOL 1.81 Option (C) is correct.

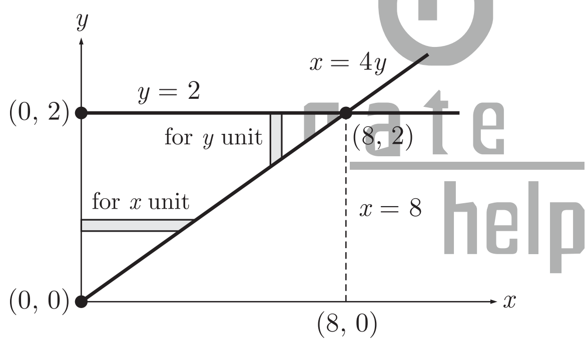
We know, from the Echelon form the rank of any matrix is equal to the Number of non zero rows.

Here order of matrix is 3×4, then, we can say that the Highest possible rank of this matrix is 3.

SOL 1.82 Option (A) is correct.

Given $$I = \int_0^8 \int_{\pi/4}^2 f(x,y)\,dydx$$

We can draw the graph from the limits of the integration, the limit of y is from $y = \frac{x}{4}$ to $y = 2$. For x the limit is $x = 0$ to $x = 8$



Here we change the order of the integration. The limit of x is 0 to 8 but we have to find the limits in the form of y then $x = 0$ to $x = 4y$ and limit of y is 0 to 2

So $$\int_0^8 \int_{x/4}^2 f(x,y)\,dydx = \int_0^2 \int_0^{4y} f(x,y)\,dxdy = \int_r^s \int_p^q f(x,y)\,dxdy$$

Comparing the limits and get

$r = 0,\ s = 2,\ p = 0,\ q = 4y$

SOL 1.83 Option (A) is correct.

Let, $$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

The characteristic equation for eigen values is given by,

$$|A-\lambda I| = 0$$

$$A = \begin{vmatrix} 5-\lambda & 0 & 0 & 0 \\ 0 & 5-\lambda & 0 & 0 \\ 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 3 & 1-\lambda \end{vmatrix} = 0$$

Solving this, we get

$$(5-\lambda)(5-\lambda)[(2-\lambda)(1-\lambda)-3] = 0$$

$$(5-\lambda)^2[2-3\lambda+\lambda^2-3] = 0$$

$$(5-\lambda)^2(\lambda^2-3\lambda-1) = 0$$

So, $(5-\lambda)^2 = 0 \Rightarrow \lambda = 5,\ 5$ and $\lambda^2-3\lambda-1 = 0$

$$\lambda = \frac{-(-3)\pm\sqrt{9+4}}{2} = \frac{3+\sqrt{13}}{2},\ \frac{3-\sqrt{13}}{2}$$

The eigen values are $\lambda = 5,\ 5,\ \frac{3+\sqrt{13}}{2},\ \frac{3-\sqrt{13}}{2}$

Let $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

be the eigen vector for the eigen value $\lambda = 5$

Then, $(A-\lambda I)X_1 = 0$

$$(A-5I)X_1 = 0$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 3 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

or $-3x_3 + x_4 = 0$

$$3x_3 - 4x_4 = 0$$

This implies that $x_3 = 0,\ x_4 = 0$

Let $x_1 = k_1$ and $x_2 = k_2$

So, eigen vector, $X_1 = \begin{bmatrix} k_1 \\ k_2 \\ 0 \\ 0 \end{bmatrix}$ where $k_1,\ k_2\ \varepsilon\ R$

SOL 1.84 Option (C) is correct.

Given : $x + y = 2$ ...(i)

$1.01x + 0.99y = b,\ db = 1$ unit ...(ii)

We have to find the change in x in the solution of the system. So reduce y

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From the equation (i) and (ii).

Multiply equation (i) by 0.99 and subtract from equation (ii)

$$1.01x + 0.99y - (0.99x + 0.99y) = b - 1.98$$

$$1.01x - 0.99x = b - 1.98$$

$$0.02x = b - 1.98$$

Differentiating both the sides, we get

$$0.02\,dx = db$$

$$dx = \frac{1}{0.02} = 50 \text{ unit} \qquad db = 1$$

SOL 1.85 Option (A) is correct.

Given, $x(u, v) = uv$

$$\frac{dx}{du} = v, \qquad \frac{dx}{dv} = u$$

And $y(u, v) = \frac{v}{u}$

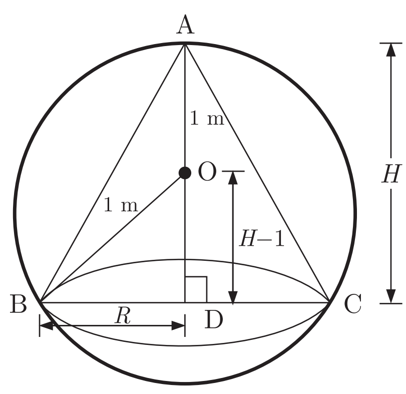
$$\frac{\partial y}{\partial u} = -\frac{v}{u^2}, \qquad \frac{\partial y}{\partial v} = \frac{1}{u}$$

We know that,

$$\phi(u, v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$\phi(u, v) = \begin{vmatrix} v & u \\ \frac{-v}{u^2} & \frac{1}{u} \end{vmatrix} = v \times \frac{1}{u} - u \times \left(-\frac{v}{u^2}\right) = \frac{v}{u} + \frac{v}{u} = \frac{2v}{u}$$

SOL 1.86 Option (D) is correct.



Given : Radius of sphere $r = 1$

Let, Radius of cone $= R$

Height of the cone $= H$

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Finding the relation between the volume and Height of the cone

From ΔOBD, $$OB^2 = OD^2 + BD^2$$

$$1 = (H-1)^2 + R^2 = H^2 + 1 - 2H + R^2$$

$$R^2 + H^2 - 2H = 0$$

$$R^2 = 2H - H^2 \qquad \text{...(i)}$$

Volume of the cone, $$V = \frac{1}{3}\pi R^2 H$$

Substitute the value of R^2 from equation (i), we get

$$V = \frac{1}{3}\pi(2H - H^2)H = \frac{1}{3}\pi(2H^2 - H^3)$$

Differentiate V w.r.t to H

$$\frac{dV}{dH} = \frac{1}{3}\pi[4H - 3H^2]$$

Again differentiate $$\frac{d^2V}{dH^2} = \frac{1}{3}\pi[4 - 6H]$$

For minimum and maximum value, using the principal of minima and maxima.

Put $\frac{dV}{dH} = 0$

$$\frac{1}{3}\pi[4H - 3H^2] = 0$$

$$H[4 - 3H] = 0 \Rightarrow H = 0 \text{ and } H = \frac{4}{3}$$

At $H = \frac{4}{3}$, $$\frac{d^2V}{dH^2} = \frac{1}{3}\pi\left[4 - 6 \times \frac{4}{3}\right] = \frac{1}{3}\pi[4 - 8] = -\frac{4}{3}\pi < 0 \qquad \text{(Maxima)}$$

And at $H = 0$, $$\frac{d^2V}{dH^2} = \frac{1}{3}\pi[4 - 0] = \frac{4}{3}\pi > 0 \qquad \text{(Minima)}$$

So, for the largest volume of cone, the value of H should be 4/3

SOL 1.87 Option (D) is correct.

Given : $$x^2\frac{dy}{dx} + 2xy = \frac{2\ln(x)}{x}$$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{2\ln(x)}{x^3}$$

Comparing this equation with the differential equation $\frac{dy}{dx} + P(y) = Q$ we have $P = \frac{2}{x}$ and $Q = \frac{2\ln(x)}{x^3}$

The integrating factor is,

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{2}{x} dx}$$

$$e^{2\ln x} = e^{\ln x^2} = x^2$$

Complete solution is written as,

$$y(I.F.) = \int Q(I.F.)\,dx + C$$

$$y(x^2) = \int \frac{2\ln x}{x^3} \times x^2\,dx + C = 2\int \underset{(II)}{\ln x} \times \underset{(I)}{\frac{1}{x}}\,dx + C \qquad \text{...(i)}$$

Integrating the value $\int \ln x \times \frac{1}{x}\,dx$ Separately

Let, $$I = \int \underset{(I)}{\ln x} \times \underset{(II)}{\frac{1}{x}}\,dx \qquad \text{...(ii)}$$

$$= \ln x \int \frac{1}{x}\,dx - \int \left\{\frac{d}{dx}(\ln x) \times \int \frac{1}{x}\,dx\right\}dx$$

$$= \ln x \ln x - \underbrace{\int \frac{1}{x} \times \ln x dx}_{I} \qquad \text{From equation(ii)}$$

$$2I = (\ln x)^2$$

or $$I = \frac{(\ln x)^2}{2} \qquad \text{...(iii)}$$

Substitute the value from equation (iii) in equation (i),

$$y(x^2) = \frac{2(\ln x)^2}{2} + C$$

$$x^2 y = (\ln x)^2 + C \qquad \text{...(iv)}$$

Given $y(1) = 0$, means at $x = 1 \quad \Rightarrow y = 0$

then $$0 = (\ln 1)^2 + C \Rightarrow C = 0$$

So from equation (iv), we get

$$x^2 y = (\ln x)^2$$

Now at $x = e$, $$y(e) = \frac{(\ln e)^2}{e^2} = \frac{1}{e^2}$$

SOL 1.88 Option (A) is correct.

Potential function of $v = x^2 yz$ at $P(1,1,1)$ is $= 1^2 \times 1 \times 1 = 1$ and at origin $O(0,0,0)$ is 0.

Thus the integral of vector function from origin to the point (1,1,1) is

$$= [x^2 yz]_P - [x^2 yz]_O$$

$$= 1 - 0 = 1$$

SOL 1.89 Option (C) is correct.

Let, $$f(x) = x^3 + 3x - 7$$

From the Newton Rapson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad \text{...(i)}$$

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We have to find the value of x_1, so put $n = 0$ in equation (i),

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(x) = x^3 + 3x - 7$$

$$f(x_0) = 1^3 + 3 \times 1 - 7 = 1 + 3 - 7 = -3 \qquad x_0 = 1$$

$$f'(x) = 3x^2 + 3$$

$$f'(x_0) = 3 \times (1)^2 + 3 = 6$$

Then, $$x_1 = 1 - \frac{(-3)}{6} = 1 + \frac{3}{6} = 1 + \frac{1}{2} = \frac{3}{2} = 1.5$$

SOL 1.90 Option (D) is correct.

We know a die has 6 faces and 6 numbers so the total number of ways

$$= 6 \times 6 = 36$$

And total ways in which sum is either 8 or 9 is 9, i.e.

(2,6),(3,6) (3,5) (4,4) (4,5) (5,4) (5,3) (6,2) (6,3)

Total number of tosses when both the 8 or 9 numbers are not come

$$= 36 - 9 = 27$$

Then probability of not coming sum 8 or 9 is, $= \frac{27}{36} = \frac{3}{4}$

SOL 1.91 Option (C) is correct.

Given : $$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = 0$$

The solution of this equation is given by,

$$y = c_1 e^{mx} + c_2 e^{nx} \qquad \text{...(i)}$$

Here m & n are the roots of ordinary differential equation

Given solution is, $$y = c_1 e^{-x} + c_2 e^{-3x} \qquad \text{...(ii)}$$

Comparing equation (i) and (ii), we get $m = -1$ and $n = -3$

Sum of roots, $$m + n = -p$$

$$-1 - 3 = -p \Rightarrow p = 4$$

and product of roots, $$mn = q$$

$$(-1)(-3) = q \Rightarrow q = 3$$

SOL 1.92 Option (C) is correct.

Given : $$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + (q+1)y = 0$$

$$[D^2 + pD + (q+1)]y = 0 \qquad \frac{d}{dx} = D$$

From the previous question, put $p = 4$ and $m = 3$

$$[D^2 + 4D + 4]y = 0 \qquad \text{...(i)}$$

The auxilliary equation of equation (i) is written as

$$m^2 + 4m + 4 = 0 \Rightarrow m = -2, -2$$

Here the roots of auxiliary equation are same then the solution is

$$y = (c_1 + c_2 x)\, e^{mx} = xe^{-2x} \qquad \begin{pmatrix} \text{Let } c_1 = 0 \\ c_2 = 1 \end{pmatrix}$$

SOL 1.93 Option (C) is correct.

Given : $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$

First differentiate x w.r.t. θ,

$$\frac{dx}{d\theta} = a[1 + \cos\theta]$$

And differentiate y w.r.t. θ

$$\frac{dy}{d\theta} = a[0 - (-\sin\theta)] = a\sin\theta$$

We know, $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \times \dfrac{d\theta}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$

Substitute the values of $\dfrac{dy}{d\theta}$ and $\dfrac{dx}{d\theta}$

$$\frac{dy}{dx} = a\sin\theta \times \frac{1}{a[1 + \cos\theta]} = \frac{\sin\theta}{1 + \cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2} \qquad \cos\theta + 1 = 2\cos^2\frac{\theta}{2}$$

SOL 1.94 Option (C) is correct.

Given : $P(0.866, 0.500, 0)$, so we can write

$$\boldsymbol{P} = 0.866\boldsymbol{i} + 0.5\boldsymbol{j} + 0\boldsymbol{k}$$

$Q = (0.259, 0.966, 0)$, so we can write

$$\boldsymbol{Q} = 0.259\boldsymbol{i} + 0.966\boldsymbol{j} + 0\boldsymbol{k}$$

For the coplanar vectors

$$\boldsymbol{P} \cdot \boldsymbol{Q} = |\boldsymbol{P}||\boldsymbol{Q}|\cos\theta$$

$$\cos\theta = \frac{\boldsymbol{P} \cdot \boldsymbol{Q}}{|\boldsymbol{P}||\boldsymbol{Q}|}$$

$$\boldsymbol{P} \cdot \boldsymbol{Q} = (0.866\boldsymbol{i} + 0.5\boldsymbol{j} + 0\boldsymbol{k}) \cdot (0.259\boldsymbol{i} + 0.966\boldsymbol{j} + 0\boldsymbol{k})$$

$$= 0.866 \times 0.259 + 0.5 \times 0.966$$

So, $$\cos\theta = \frac{0.866 \times 0.259 + 0.5 \times 0.966}{\sqrt{(0.866)^2 + (0.5)^2} + \sqrt{(0.259)^2 + (0.966)^2}}$$

$$= \frac{0.22429 + 0.483}{\sqrt{0.99} \times \sqrt{1.001}} = \frac{0.70729}{\sqrt{0.99} \times \sqrt{1.001}} = 0.707$$

$$\theta = \cos^{-1}(0.707) = 45^\circ$$

SOL 1.95 Option (B) is correct.

Let $$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

We know that the sum of the eigen value of a matrix is equal to the sum of the diagonal elements of the matrix
So, the sum of eigen values is,

$$1+5+1 = 7$$

SOL 1.96 Option (D) is correct.
Given : Total number of cards = 52 and two cards are drawn at random.
Number of kings in playing cards = 4
So the probability that both cards will be king is given by,

$$P = \frac{^4C_1}{^{52}C_1} \times \frac{^3C_1}{^{51}C_1} = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221} \qquad {}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$$

SOL 1.97 Option (B) is correct.

Given : $$U(t-a) = \begin{cases} 0, & \text{for } t < a \\ 1, & \text{for } t \geq a \end{cases}$$

From the definition of Laplace Transform

$$\mathcal{L}[F(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$\mathcal{L}[U(t-a)] = \int_0^\infty e^{-st}\,U(t-a)\,dt$$

$$= \int_0^a e^{-st}(0) + \int_a^\infty e^{-st}(1)\,dt = 0 + \int_a^\infty e^{-st}\,dt$$

$$\mathcal{L}[U(t-a)] = \left[\frac{e^{-st}}{-s}\right]_a^\infty = 0 - \left[\frac{e^{-as}}{-s}\right] = \frac{e^{-as}}{s}$$

SOL 1.98 Option (D) is correct.
First we have to make the table from the given data

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

Take $x_0 = 0$ and $h = 1$

Then $$P = \frac{x - x_0}{h} = x$$

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From Newton's forward Formula

$$f(x) = f(x_0) + \frac{P}{\underline{|1}}\Delta f(0) + \frac{P(P-1)}{\underline{|2}}\Delta^2 f(0) + \frac{P(P-1)(P-2)}{\underline{|3}}\Delta^3 f(0)$$

$$= f(0) + x\Delta f(0) + \frac{x(x-1)}{2}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{6}\Delta^3 f(0)$$

$$= 1 + x(1) + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{6}(12)$$

$$= 1 + x - x(x-1) + 2x(x-1)(x-2)$$

$$f(x) = 2x^3 - 7x^2 + 6x + 1$$

SOL 1.99 Option (A) is correct.

Given : $$V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2 \sin\phi\, dr d\phi d\theta$$

First integrating the term of r, we get

$$V = \int_0^{2\pi}\int_0^{\pi/3}\left[\frac{r^3}{3}\right]_0^1 \sin\phi\, d\phi d\theta = \int_0^{2\pi}\int_0^{\pi/3}\frac{1}{3}\sin\phi\, d\phi d\theta$$

Integrating the term of ϕ, we have

$$V = \frac{1}{3}\int_0^{2\pi}[-\cos\phi]_0^{\pi/3} d\theta$$

$$= -\frac{1}{3}\int_0^{2\pi}\left[\cos\frac{\pi}{3} - \cos 0\right] d\theta = -\frac{1}{3}\int_0^{2\pi}\left[\frac{1}{2} - 1\right] d\theta$$

$$= -\frac{1}{3}\int_0^{2\pi}\left(-\frac{1}{2}\right) d\theta = -\frac{1}{3}\times\left(-\frac{1}{2}\right)\int_0^{2\pi} d\theta$$

Now, integrating the term of θ, we have

$$V = \frac{1}{6}[\theta]_0^{2\pi} = \frac{1}{6}[2\pi - 0] = \frac{\pi}{3}$$

SOL 1.100 Option (A) is correct.

Let, $$A = \begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

For singularity of the matrix $|A| = 0$

$$\begin{vmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{vmatrix} = 0$$

$$8[0 - 2\times 6] - x[0 - 24] + 0[24 - 0] = 0$$

$$8\times(-12) + 24x = 0$$

$$-96 + 24x = 0 \Rightarrow x = \frac{96}{24} = 4$$

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SOL 1.101 Option (A) is correct

Let,
$$f(x) = \lim_{x \to 0} \frac{\sin^2 x}{x} = \lim_{x \to 0} \frac{\sin^2 x}{x} \times \frac{x}{x}$$
$$= \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2 \times x \qquad \lim_{x \to 0} \frac{\sin x}{x} = 1$$
$$= (1)^2 \times 0 = 0$$

Alternative :

Let
$$f(x) = \lim_{x \to 0} \frac{\sin^2 x}{x} \qquad \left[\frac{0}{0} \text{form}\right]$$
$$f(x) = \lim_{x \to 0} \frac{2 \sin x \cos x}{1} \qquad \text{Apply L-Hospital rule}$$
$$= \lim_{x \to 0} \frac{\sin 2x}{1} = \frac{\sin 0}{1} = 0$$

SOL 1.102 Option (D) is correct.

Accuracy of Simpson's rule quadrature is $O(h^5)$

SOL 1.103 Option (C) is correct.

Let,
$$A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

The characteristic equation for the eigen value is given by,
$$|A - \lambda I| = 0 \qquad I = \text{Identity matrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\left|\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$
$$\begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = 0$$
$$(4-\lambda)(4-\lambda) - 1 = 0$$
$$(4-\lambda)^2 - 1 = 0$$
$$\lambda^2 - 8\lambda + 15 = 0$$

Solving above equation, we get
$$\lambda = 5, 3$$

SOL 1.104 Option (C) is correct.

Given :
$$x + 2y + z = 6$$
$$2x + y + 2z = 6$$
$$x + y + z = 5$$

Comparing to $Ax = B$,we get

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$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 6 \\ 5 \end{bmatrix}$$

Write the system of simultaneous equations in the form of Augmented matrix,

$$[A:B] = \begin{bmatrix} 1 & 2 & 1 & : & 6 \\ 2 & 1 & 2 & : & 6 \\ 1 & 1 & 1 & : & 5 \end{bmatrix} \qquad R_2 \to R_2 - 2R_1 \text{ and } R_3 \to 2R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 6 \\ 0 & -3 & 0 & : & -6 \\ 0 & 1 & 0 & : & 4 \end{bmatrix} \qquad R_3 \to 3R_3 + R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & : & 6 \\ 0 & -3 & 0 & : & -6 \\ 0 & 0 & 0 & : & 6 \end{bmatrix}$$

It is a echelon form of matrix.

Since $\rho[A] = 2$ and $\rho[A:B] = 3$

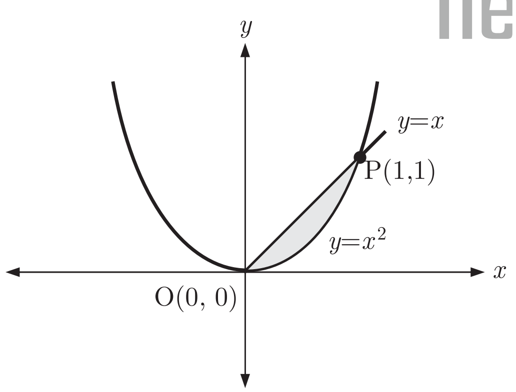
$$\rho[A] \neq \rho[A:B]$$

So, the system has no solution and system is inconsistent.

SOL 1.105 Option (B) is correct.

Given : $y = x^2$ and $y = x$.

The shaded area shows the area, which is bounded by the both curves.



Solving given equation, we get the intersection points as,

In $y = x^2$ putting $y = x$ we have $x = x^2$ or $x^2 - x = 0$ which gives $x = 0,1$

Then from $y = x$ we can see that curve $y = x^2$ and $y = x$ intersects at point (0,0) and (1,1). So, the area bounded by both the curves is

$$A = \int_{x=0}^{x=1} \int_{y=x}^{y=x^2} dydx = \int_{x=0}^{x=1} dx \int_{y=x}^{y=x^2} dy = \int_{x=0}^{x=1} dx[y]_x^{x^2}$$

$$= \int_{x=0}^{x=1} (x^2 - x) = \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6} = \frac{1}{6}\text{unit}^2$$

Area is never negative

SOL 1.106 Option (A) is correct.

$$\frac{dy}{dx} + y^2 = 0$$

$$\frac{dy}{dx} = -y^2$$

$$-\frac{dy}{y^2} = dx$$

Integrating both the sides, we have

$$-\int \frac{dy}{y^2} = \int dx$$

$$y^{-1} = x + c \Rightarrow y = \frac{1}{x+c}$$

SOL 1.107 Option (C) is correct.

Given : $\boldsymbol{F} = x\boldsymbol{i} - y\boldsymbol{j}$

First Check divergency, for divergence,

$$\text{Grade } \boldsymbol{F} = \nabla \cdot \boldsymbol{F} = \left[\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right] \cdot [x\boldsymbol{i} - y\boldsymbol{j}] = 1 - 1 = 0$$

So we can say that $\boldsymbol{F}$ is divergence free.

Now checking the irrationalit;. For irritation the curl $\boldsymbol{F} = 0$

$$\text{Curl } \boldsymbol{F} = \nabla \times \boldsymbol{F} = \left[\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right] \times [x\boldsymbol{i} - y\boldsymbol{j}]$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & 0 \end{vmatrix} = \boldsymbol{i}[0-0] - \boldsymbol{j}[0-0] + \boldsymbol{k}[0-0] = 0$$

So, vector field is irrotational. We can say that the vector field is divergence free and irrotational.

SOL 1.108 Option (B) is correct.

Let $f(t) = \sin \omega t$

From the definition of Laplace transformation

$$\mathcal{L}[F(t)] = \int_0^\infty e^{-st} f(t)\, dt = \int_0^\infty e^{-st} \sin \omega t dt$$

$$= \int_0^\infty e^{-st} \left(\frac{e^{i\omega t} - e^{-i\omega t}}{2i}\right) dt$$

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$$\sin\omega t = \frac{e^{i\omega t} - e^{-i\omega t}}{2i} = \frac{1}{2i}\int_0^\infty (e^{-st}e^{i\omega t} - e^{-st}e^{-i\omega t})\,dt$$

$$= \frac{1}{2i}\int_0^\infty [e^{(-s+i\omega)t} - e^{-(s+i\omega)t}]\,dt$$

Integrating above equation, we get

$$\sin\omega t = \frac{1}{2i}\left[\frac{e^{(-s+i\omega)t}}{-s+i\omega} - \frac{e^{-(s+i\omega)t}}{-(s+i\omega)}\right]_0^\infty$$

$$= \frac{1}{2i}\left[\frac{e^{(-s+i\omega)t}}{-s+i\omega} + \frac{e^{-(s+i\omega)t}}{(s+i\omega)}\right]_0^\infty$$

Substitute the limits, we get

$$\sin\omega t = \frac{1}{2i}\left[0+0-\left(\frac{e^0}{(-s+i\omega)} + \frac{e^{-0}}{s+i\omega}\right)\right]$$

$$= -\frac{1}{2i}\left[\frac{s+i\omega+i\omega-s}{(-s+i\omega)(s+i\omega)}\right]$$

$$= -\frac{1}{2i}\times\frac{2i\omega}{(i\omega)^2 - s^2} = \frac{-\omega}{-\omega^2 - s^2} = \frac{\omega}{\omega^2+s^2}$$

Alternative :

From the definition of Laplace transformation

$$\mathcal{L}[F(t)] = \int_0^\infty e^{-st}\sin\omega t\,dt$$

We know $\int e^{at}\sin bt\,dt = \frac{e^{at}}{a^2+b^2}[a\sin bt - b\cos bt]$ $\quad\begin{pmatrix} a = -s \text{ and} \\ b = \omega \end{pmatrix}$

Then, $$\mathcal{L}[\sin\omega t] = \left[\frac{e^{-st}}{s^2+\omega^2}(-s\sin\omega t - \omega\cos\omega t)\right]_0^\infty$$

$$= \left[\frac{e^{-\infty}}{s^2+\omega^2}(-s\sin\infty - \omega\cos\infty)\right] - \left[\frac{e^{-0}}{s^2+\omega^2}(-s\sin 0 - \omega\cos 0)\right]$$

$$= 0 - \frac{1}{s^2+\omega^2}[0-\omega] = -\frac{1}{s^2+\omega^2}(-\omega)$$

$$\mathcal{L}[\sin\omega t] = \frac{\omega}{s^2+\omega^2}$$

SOL 1.109 Option (D) is correct.

Given : black balls = 5, Red balls = 5, Total balls=10

Here, two balls are picked from the box randomly one after the other without replacement. So the probability of both the balls are red is

$$P = \frac{{}^5C_0 \times {}^5C_2}{{}^{10}C_2} = \frac{\frac{5!}{0!\times 5!}\times\frac{5!}{3!2!}}{\frac{10!}{3!2!}} = \frac{1\times 10}{45} = \frac{10}{45} = \frac{2}{9} \qquad {}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$$

Alternate Method :

Given : Black balls = 5,

$$\text{Red balls} = 5$$
$$\text{Total balls} = 10$$

The probability of drawing a red bell,

$$P_1 = \frac{5}{10} = \frac{1}{2}$$

If ball is not replaced, then box contains 9 balls.
So, probability of drawing the next red ball from the box.

$$P_2 = \frac{4}{9}$$

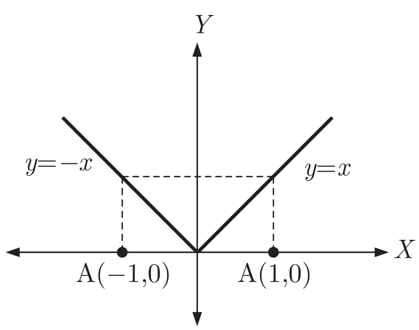
Hence, probability for both the balls being red is,

$$P = P_1 \times P_2 = \frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$$

SOL 1.110 Option (A) is correct.
We know that a dice has 6 faces and 6 numbers so the total number of cases (outcomes) $= 6 \times 6 = 36$
And total ways in which sum of the numbers on the dices is eight,
(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)
So, the probability that the sum of the numbers eight is,

$$p = \frac{5}{36}$$

SOL 1.111 Option (D) is correct.
We have to draw the graph on x-y axis from the given functions.



$$f(x) = \begin{cases} -x & x \le -1 \\ 0 & x = 0 \\ x & x \ge 1 \end{cases}$$

It clearly shows that $f(x)$ is differential at $x = -1$, $x = 0$ and $x = 1$, i.e. in the domain $[-1, 1]$.
So, (a), (b) and (c) are differential and $f(x)$ is maximum at $(x, -x)$.

SOL 1.112 Option (B) is correct.
If the scatter diagram indicates some relationship between two variables X

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and Y, then the dots of the scatter diagram will be concentrated round a curve. This curve is called the curve of regression.
Regression analysis is used for estimating the unknown values of one variable corresponding to the known value of another variable.

SOL 1.113 Option (B) is correct.

Given :
$$3x + 2y + z = 4$$
$$x - y + z = 2$$
$$-2x + 2z = 5$$

The Augmented matrix of the given system of equation is

$$[A:B] = \begin{bmatrix} 3 & 2 & 1 & : & 4 \\ 1 & -1 & 1 & : & 2 \\ -2 & 0 & 2 & : & 5 \end{bmatrix} \quad R_3 \to R_3 + 2R_2,\ R_2 \to R_2 - R_1$$

$$\sim \begin{bmatrix} 3 & 2 & 1 & : & 4 \\ -2 & -3 & 0 & : & -2 \\ 0 & -2 & 4 & : & 9 \end{bmatrix}$$

Here $\rho[A:B] = \rho[A] = 3 = n$ (number of unknown)
Then the system of equation has a unique solution.

SOL 1.114 Option (B) is correct.

Given :
$$f(x,y) = 2x^2 + 2xy - y^3$$

Partially differentiate this function w.r.t x and y,

$$\frac{\partial f}{\partial x} = 4x + 2y, \quad \frac{\partial f}{\partial y} = 2x - 3y^2$$

For the stationary point of the function, put $\partial f/\partial x$ and $\partial f/\partial y$ equal to zero.

$$\frac{\partial f}{\partial x} = 4x + 2y = 0 \qquad \Rightarrow \quad 2x + y = 0 \qquad \text{...(i)}$$

and
$$\frac{\partial f}{\partial y} = 2x - 3y^2 = 0 \qquad \Rightarrow \quad 2x - 3y^2 = 0 \qquad \text{...(ii)}$$

From equation (i), $y = -2x$ substitute in equation (ii),

$$2x - 3(-2x)^2 = 0$$
$$2x - 3 \times 4x^2 = 0$$
$$6x^2 - x = 0 \Rightarrow x = 0, \frac{1}{6}$$

From equation (i),

For $x = 0$, $\quad y = -2 \times (0) = 0$

and for $x = \frac{1}{6}$, $\quad y = -2 \times \frac{1}{6} = -\frac{1}{3}$

So, two stationary point at $(0,0)$ and $\left(\frac{1}{6}, -\frac{1}{3}\right)$

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SOL 1.115 Option (B) is correct.

$$\text{Sample space} = \begin{matrix} (1,1), & (1,2) & \dots & (1,8) \\ (2,1), & (2,2) & \dots & (2,8) \\ (3,1), & (3,2) & \dots & (3,8) \\ \vdots & \vdots & \vdots & \vdots \\ (8,1), & (8,2) & \dots & (8,8) \end{matrix}$$

$$\text{Total number of sample space} = 8 \times 8 = 64$$

Now, the favourable cases when Manish will arrive late at D

$$= (6,8),\ (8,6)\dots(8,8)$$

$$\text{Total number of favourable cases} = 13$$

So,
$$\text{Probability} = \frac{\text{Total number of favourable cases}}{\text{Totol number of sample space}}$$

$$= \frac{13}{64}$$

SOL 1.116 Option (B) is correct.

Divergence is defined as $\nabla \cdot r$

where
$$r = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

and
$$\nabla = \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}$$

So,
$$\nabla \cdot r = \left(\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right) \cdot (x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k})$$

$$\nabla \cdot r = 1 + 1 + 1 = 3$$

SOL 1.117 Option (B) is correct.

Given :
$$x + y = 2$$
$$2x + 2y = 5$$

The Augmented matrix of the given system of equations is

$$[A:B] = \begin{bmatrix} 1 & 1 & : & 2 \\ 2 & 2 & : & 5 \end{bmatrix}$$

Applying row operation, $R_2 \to R_2 - 2R_1$

$$[A:B] = \begin{bmatrix} 1 & 1 & : & 2 \\ 0 & 0 & : & 1 \end{bmatrix}$$

$$\rho[A] = 1 \neq \rho[A:B] = 2$$

So, the system has no solution.

SOL 1.118 Option (D) is correct.

Given :
$$f(x) = |x|$$

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$Lf'(x) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \to 0} \frac{-(-h)}{-h} - 0 = -1$$

$$Rf'(x) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{h-0}{h} = 1$$

Since $Lf'(0) \neq Rf'(0)$

So, derivative of $f(x)$ at $x = 0$ does not exist.

SOL 1.119 Option (A) is correct.

The surface integral of the normal component of a vector function F taken around a closed surface S is equal to the integral of the divergence of F taken over the volume V enclosed by the surface S.

Mathematically $$\iint_S F \cdot \boldsymbol{n} dS = \iiint_V \text{div } \boldsymbol{F} dv$$

So, Gauss divergence theorem relates surface integrals to volume integrals.

SOL 1.120 Option (A) is correct.

Given : $$f(x) = \frac{x^3}{3} - x$$

$$f'(x) = x^2 - 1$$

$$f''(x) = 2x$$

Using the principle of maxima – minima and put $f'(x) = 0$

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

Hence at $x = -1$, $f''(x) = -2 < 0$ (Maxima)

at $x = 1$, $f''(x) = 2 > 0$ (Minima)

So, $f(x)$ is minimum at $x = 1$

SOL 1.121 Option (B) is correct.

Let $$A = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}, B = \begin{bmatrix} a_2 & b_2 & c_2 \end{bmatrix}$$

$$C = AB$$

Let $$= \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \times \begin{bmatrix} a_2 & b_2 & c_2 \end{bmatrix} = \begin{bmatrix} a_1 a_2 & a_1 b_2 & a_1 c_2 \\ b_1 a_2 & b_1 b_2 & b_1 c_2 \\ c_1 a_2 & c_1 b_2 & c_1 c_2 \end{bmatrix}$$

The 3×3 minor of this matrix is zero and all the 2×2 minors are also zero. So the rank of this matrix is 1.

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$$\rho[C] = 1$$

SOL 1.122 Option (D) is correct.

In a coin probability of getting head $p = \frac{1}{2}$ and probability of getting tail,

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

When unbiased coin is tossed three times, then total possibilities are

H H H
H H T
H T H
T H H
H T T
T T H
T H T
T T T

From these cases, there are three cases, when head comes exactly two times. So, the probability of getting head exactly two times, when coin is tossed 3 times is,

$$P = {}^3C_2 (p)^2 (q)^1 = 3 \times \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{3}{8}$$

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